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UNDERWATER EJECTOR PROPULSION THEORY AND APPLICATIONS.(U)

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FLIGHT DYNAMICS RESEARCH CORPORATION,
VAN NUYS, CALIFORNIA

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UNDERWATER EJECTOR PROPULSION
THEORY AND APPLICATIONS

by
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20 Abstract

The fundamentals of ejector propulsion are described and a theoretical treatment of the flow through an ejector is presented in detail.

The theory considers the influence of the injestion of boundary layer (or wake) fluids in the ejector and/or its pump, and the limits of performance due to cavitation in the ejector.

Methods for the evaluation of the propulsive efficiency, thrust augmentation, and relative power of ejectors are derived from fundamental laws of fluid mechanics.

Comparisons of single-stage ejectors with propellers and free jet propulsion systems are made on the basis of propulsive efficiency.

Procedures for the evaluation of multi-stage ejector performance are derived and examples are presented to illustrate the performance of two-stage ejectors under stationary and translating conditions.

CONTENTS

LIST OF SYMBOLS

A	Area
a	Jet area
c	Chord
C	Coefficient
F	Thrust (net)
P	Power, Stagnation pressure
p	Pressure
Q	Flow rate
q	Dynamic pressure
R_P	Relative power
t	Thickness
U	Velocity
V	Velocity of jet
X	Lateral dimension
α	Inlet area ratio (A_2/a)
δ	Diffuser area ratio
ρ	Mass density
η	Propulsive efficiency
ΔP	Pump, jet velocity pressure rise
λ_n	Velocity ratio (U_n/V_{j1})
ϕ	Thrust augmentation

<u>Subscripts</u>

∞	Undisturbed flow
1, 2, 3	Ejector stations
c	Critical
d	Section drag
D	Total drag
e	Ejector inlet
eff	Effective
ej	Ejector
F	Thrust
j	Jet
L	Leaving loss
p	Pump inlet, Primary
ref	Reference jet
o	Unaugmented, Stagnation
T	Total

SECTION I
INTRODUCTION

Ejectors are commonly considered as thrust augmenters, and their performance is most generally described in terms of a quantity called thrust augmentation. In general, thrust augmentation refers to the ratio of the net thrust (gross thrust minus ram drag), to the net thrust of a free, or unaugmented jet, which utilizes the same power supply, operating at the same point of its characteristic curve.

This definition is responsible for the concept that ejectors are devices which can be added to an existing system, to supplement its thrust.

While this concept has validity for some applications, it has been recognized that an ejector has a more general utility. In fact, the proper use of ejector thrust depends upon the optimization of the entire propulsion system, to achieve the desired objectives.

Use of an ejector as a supplement to an already existing jet propulsion system implies the use of the power supply and pump at the characteristic point which had been selected for free (unaugmented) jet operation. In most designs this would not correspond to the optimal characteristics for the ejector.

For example, to achieve a given thrust at a given speed, the ejector requires a considerably smaller power input than does a free jet. This can result in a smaller power supply, smaller fuel consumption rate, smaller weight and size of machinery, and many other related advantages for the ejector system.

For these reasons, this document has reported the ejector performance in terms of three criteria.

First, the thrust augmentation is presented as the ratio of the net thrust of the ejector system to the net thrust available from a free jet which is driven by the same power supply, operating at the same characteristic point.

Second, the propulsive efficiency, defined in the conventional manner as the ratio of propulsive power to jet power, is used as a comparison of the ability of an ejector to utilize jet energy for propulsion purposes.

Third, the relative power is presented as the ratio of the jet power required by an ejector, to the jet power required by a free jet of equal thrust. This definition implies the use of a power supply system for the ejector, which is different from that for the free jet. This new concept is most useful in the determination of the relative merits of free and augmented jets, and is discussed in detail in this document.

The three performance criteria are all useful in the selection of the ejector geometry, and pump characteristics required to achieve the optimal performance in terms of the vehicle's operational condition and mission.

The document describes the fundamental laws related to ejector phenomena, and discusses some of the design problems and recent advances which provide a basis for the achievement of high performance with small ejectors.

The development of the expressions for the performance criteria in terms of the ejector's geometry and power supply characteristics, from the fundamental energy, momentum and mass conservation laws are presented in detail. The limitations due to the onset of cavitation are described and considered in the mathematical derivation of the performance criteria.

The use of boundary layer fluid in the ejector and/or in the pump is also considered, and mathematical relationships for the performance criteria are presented in terms of the average velocity of the boundary layer fluid upstream of the inlets to the ejector and pump.

The drag force due to skin friction on the internal and external surfaces of the ejector is evaluated, and a correction term to the thrust augmentation is derived.

Some special applications for ejector thrust are described in detail, and the performance as a function of the power supply characteristics is presented to illustrate the methods for using the basic equations. The relative size of ejectors, compared to the size of free jets and propellers, which produce the same thrust, is discussed in connection with the special examples.

Comparisons of the propulsive efficiencies of propellers, jets and ejectors are discussed and some curves are presented which illustrate the relative merits of these three types of propulsion systems, as a function of the jet velocity ratio and the pressure rise of each device.

Methods for the calculation of the performance of multi-stage ejectors are described, and some examples of their performance are presented.

The equations for the performance criteria were developed for conventionsl solid diffuser ejectors due to the fact that the method for analysis of a jet diffuser ejector in motion is not yet available. However, it is contemplated that the jet diffuser ejector would be utilized in any practical application in view of the size and drag benefits which are attributable to the use of jet diffusion instead of solid diffusion.

SECTION II

FUNDAMENTALS OF EJECTOR TECHNOLOGY

1. FLOW CONSIDERATIONS

A thrust augmenting ejector is basically a duct, surrounding a fluid jet or jets (primary jets). The primary jet kinetic energy is utilized to produce a thrust force in excess of that of the thrust of the primary jets acting as free (unaugmented) jets.

The "excess" force developed by the ejector results from the presence of the ejector duct in the field of the entrained, or induced flow. This entrained flow results from the viscous interaction (mixing) of the induced fluid and the primary jet fluid.

A free jet will entrain a flow from its environment, as illustrated on Figure 1. This entrainment however, will not produce additional thrust, since in the absence of a solid surface, no external force can be exerted upon the fluid system after leaving the nozzle.

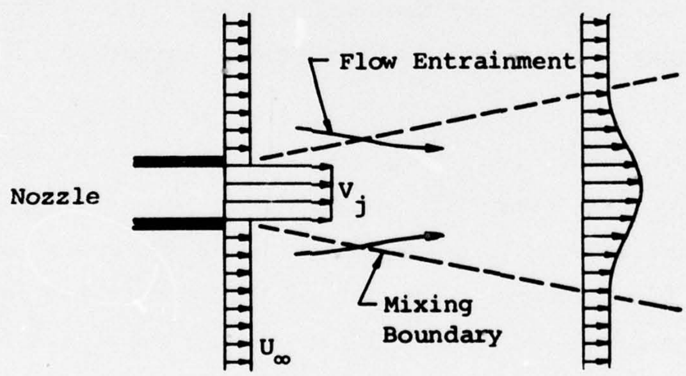


Figure 1 FREE JET

Newton's Second Law states that the rate of momentum change of a system is equal to the sum of the external forces acting on the system. Thus a free jet, which discharges into the environment in the absence of any solid surfaces, can exert forces only upon the ambient fluid. These forces which are the result of viscous interaction between the jet fluid and the ambient fluid are internal to the system comprised of these fluids. Thus according to Newton, the momentum of the system (jet plus entrained fluids) must be constant and equal to the sum of the momentum of the jet fluid leaving the nozzle, plus the momentum of the entrained fluid in its undisturbed state.

If a surface (or duct) is properly located within the entrained flow, as illustrated on Figure 2, the motion resulting from mixing, can produce a momentum increment, since forces can be applied to the fluid by the duct.

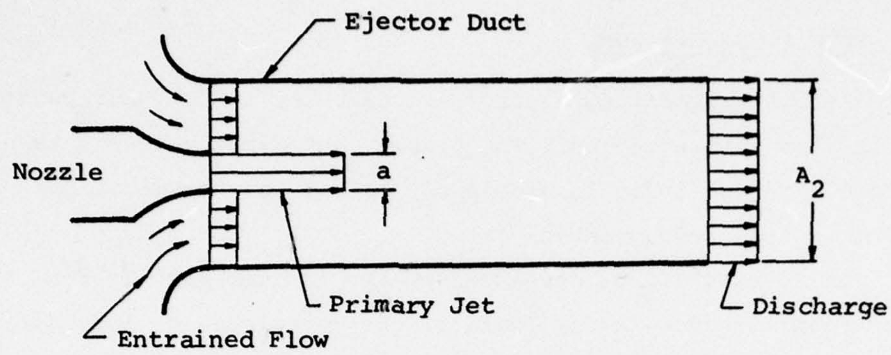


Figure 2. A SIMPLE THRUST AUGMENTER

According to Newton's Third Law, the force on the duct must be equal and opposite to the force on the fluid. Thus an increase in the momentum of the fluid (towards the right on Figure 2), must be balanced by a force on the duct (towards the left on Figure 2).

The magnitude of the momentum change, which is equal to the force on the duct, depends upon several important features of the duct design. These include:

a) The ratio (A_2/a), the inlet area ratio. Obviously, as the cross-section of the ejector duct is increased, the quantity of ambient fluid entering its inlet will increase (other parameters remaining constant).

b) The degree of mixing of the primary and induced fluid. Since the mixing of primary and induced fluids is the mechanism for the transfer of energy and momentum to the entrained fluid, it is imperative that the process of mixing be carried out to the maximum extent possible within the region of the flow field where the pressure is sub-ambient. Mixing at ambient pressure, outside of the ejector duct does not produce thrust, as indicated in the previous section.

c) The shape of the ejector's duct. The inlet and diffuser sections of the ejector play a dominant role in the production of thrust. For example, the inlet shape must be such that the incoming flow is captured with a minimal disturbance while exposing a minimal frontal area and wetted surface to avoid excessive drag.

In addition, the use of a diverging duct cross-section (a diffuser), as illustrated on Figure 3, results in a pressure reduction at the narrow cross-section (throat) of the ejector, compared to the ambient pressure at the large end of the ejector's diffuser.

This pressure reduction at the throat of the ejector results in an increase of the entrainment ratio, and as indicated on Figure 4, also provides a large increase in thrust augmentation with increasing diffuser area ratio. Although Figure 4 represents the influence of diffuser area ratio for a stationary ejector, similar relationships exist for a translating ejector. The ejector in motion in its thrust direction will be discussed in more detail in later sections of this document.

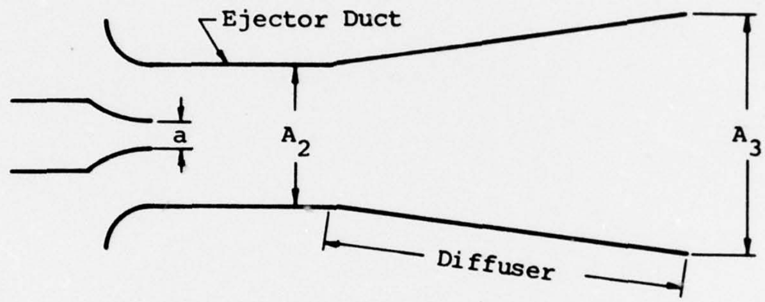


Figure 3 SIMPLE EJECTOR WITH DIFFUSER

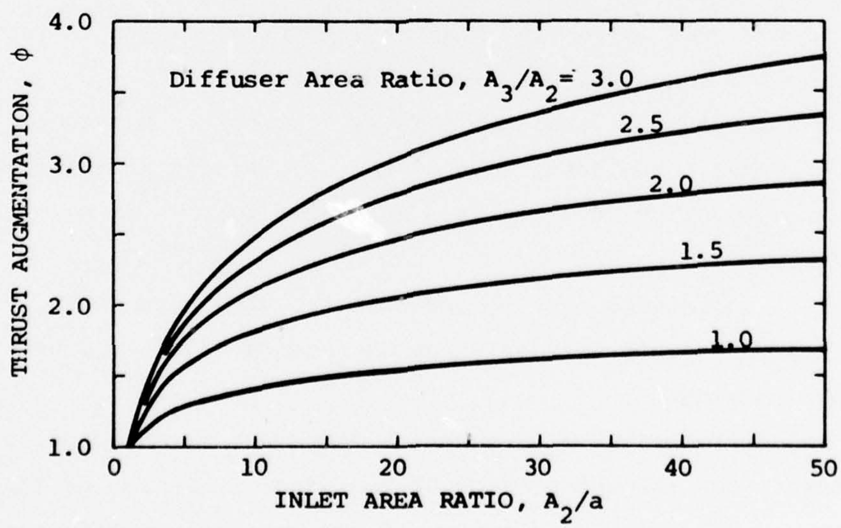


Figure 4 INFLUENCE OF INLET AND DIFFUSER AREA RATIO ON EJECTOR THRUST AUGMENTATION

2. DESIGN CONSIDERATIONS

Ejectors designed as illustrated on Figure 3, require excessively large dimensions in the thrust direction, to provide the space required for mixing and diffusion. This fact, more than any other aspect of ejector technology has delayed the practical utilization of ejectors as thrust augmenters.

Recent research and development efforts have resulted in major advances in size reduction for these processes (Reference 1).

a) Mixing

Mixing of the injected, high stagnation pressure, primary fluid with the induced, ambient fluid, within the ejector, can be accelerated by means of the use of specially designed nozzles, oriented at an angle (greater than zero) to the ejector's thrust axis, as illustrated on Figure 5.

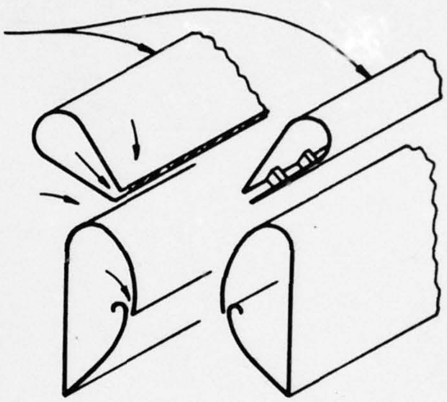


Figure 5. ANGULAR PRIMARY NOZZLES WITH SPACERS

Spacers (or separators) which divide the primary flow into a series of jets, instead of one continuous jet sheet, in combination with the curved streamlines, resulting from angular injection, result in large reductions of the required length of the mixing section.

These are two techniques utilized to achieve a reduction of ejector mixing length. Other techniques, such as the Hypermixing Nozzle (Reference 2) have been used by other investigators with some success in accelerating the mixing process.

Such techniques must be incorporated with care to avoid excessive internal and external nozzle losses, which can easily negate any benefit resulting from improved mixing.

The diffuser, a very essential component for high performance ejectors, represents the largest portion of the ejector. Design of small, highly efficient ejectors require the development of non-separating diffusers having wide angles, and thereby short lengths for any given area ratio. One such diffuser (the jet diffuser) has been developed by FDRC.

b) <u>Diffusion</u>

As illustrated on Figure 4, the theoretical thrust augmentation of an ejector increases with increasing diffuser area ratio. This theoretical performance advantage is derived from a lossless divergence of the streamlines, and the influence of skin friction and flow separation can result in an adverse effect. Excessive divergence angles, or surfaces which diverge at small angles, but which must be long in the flow direction to achieve large area ratios, can result in excessive skin friction or flow separation. These influences result in <u>effective</u> diffuser area ratios which are smaller than the geometric area ratio, as indicated on Figure 6.

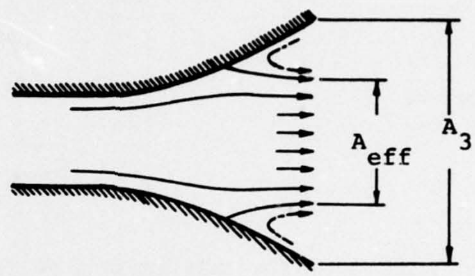


Figure 6
SEPARATED DIFFUSER

To avoid the separation, and to permit use of very large diffuser divergence angles, a diffuser jet is placed within the diffuser, at a region where separation has not yet occurred. The diffuser jet is located within the curved solid surfaces as illustrated on Figure 7. Experiments with this type of diffuser (Reference 1) have been performed with curved surfaces terminating at an angle of 45 degrees, to the axis of symmetry, without separation.

The diffuser jet, if properly designed, can continue beyond the solid surface, to form a curved fluid wall (jet diffuser). This stream can sustain a pressure difference, and therefore can act as a diffuser surface.

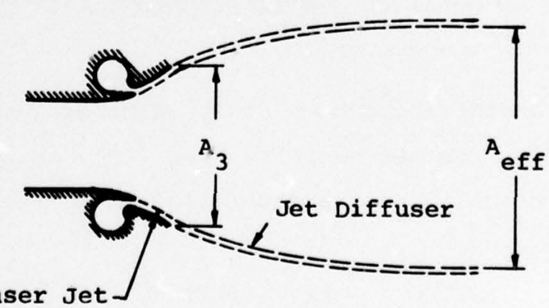


Figure 7
JET DIFFUSER

Tests (Reference 1) have indicated that, with a jet diffuser, the effective diffuser area ratio can exceed that of the geometric area ratio of the solid surfaces. Diffusers designed in this manner can achieve a given area ratio with a length which is less than 25% of that required for conventional solid diffusers.

3. POWER AND THRUST CONSIDERATIONS

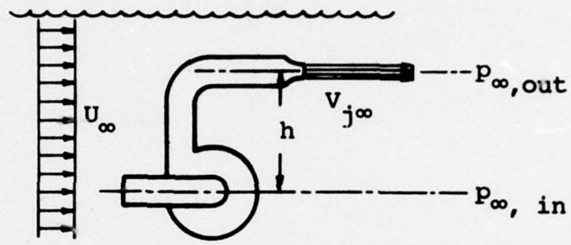
a) Free Jet

Jet thrust is developed by the expansion of a previously pressurized fluid flow through a nozzle.

For the purpose of this discussion, the fluid is considered to be incompressible, injested from the environment, and passed through a pump for pressurization.

Consider first a free (unaugmented) jet, discharging to the ambient pressure as illustrated on Figure 8.

Figure 8
FREE JET



The flow through the system can be described with the aid of Bernoulli's Equation, if the exit flow is located at a distance (h) above the inlet flow, as follows;

$$p_{\infty,in} + (\rho/2)U_{\infty}^2 + \Delta P = p_{\infty,out} + (\rho/2)V_{j\infty}^2 + \rho gh \tag{1}$$

where, ΔP represents the jet velocity pressure rise and

$(P_{\infty,in} - P_{\infty,out})$ represents the difference in ambient pressure between the inlet and outlet of the system, which for a completely submerged system is equal to ρgh (the hydrostatic head resulting from the weight of water in the system).

Considering a completely submerged system, Equation 1 can be rearranged to determine the jet velocity head (ΔP), in terms of the velocities; a form which will be useful in the determination of the jet power. Solving for (ΔP), Equation 1 becomes,

$$\Delta P = (\rho/2)(V_{j\infty}^2 - U_{\infty}^2) \tag{2}$$

where $V_{j\infty}$ is used to indicate that the jet velocity is the result of a discharge to ambient pressure, to differentiate this velocity from that which would result from a discharge to some other pressure, as would occur if the jet were discharged into an ejector.

The power required to drive a jet (exclusive of duct and machinery losses and static head), can be expressed as the rate of change of flow kinetic energy, or one half the mass flow rate times the change of the square of the velocities. Mathematically this is expressed as,

$$P_j = (\rho/2)AV_{j\infty}(V_{j\infty}^2 - U_\infty^2) = Q(\Delta P) \tag{3}$$

where

$Q = AV_{j\infty}$ = volume flow rate of working fluid

A = area of the jet nozzle exit

The net thrust force (F) exerted on the vehicle by the propulsion system is exactly equal to the momentum flux imparted to the working fluid by the system. This can be expressed mathematically as the difference between the jet discharge momentum flux and the intake fluid momentum flux (at free-stream conditions) or,

$$F = \dot{m}(V_{j\infty} - U_\infty) = \rho AV_{j\infty}(V_{j\infty} - U_\infty) \tag{4}$$

since the change of momentum flux is equal to the mass flow rate times the velocity change resulting from the passage through the propulsion system.

The ratio of net thrust (F) to the jet power (P_j) can be found from Equations 3 and 4 as,

$$F/P_j = 2/(V_{j\infty} + U_\infty) \tag{5}$$

which illustrates the influence of jet velocity upon the thrust/power ratio.

Larger values of thrust/power require smaller jet velocities. Unfortunately, as the velocity $V_{j\infty}$ is decreased, the area A must be increased to provide a given thrust. This implies large size equipment (pumps or compressors) which become prohibitive in terms of space, drag and cost, for an unaugmented jet.

A more detailed examination of the distribution of jet power (P_j), between the useful thrust power (P_F), and the power lost to the environment as heat (P_L), can proceed by writing the mathematical expressions for these quantities.

First, the propulsive or thrust power can be expressed as the thrust multiplied by the vehicle velocity.

$$P_F = F \times U_\infty = \rho AV_{j\infty}(V_{j\infty} - U_\infty)U_\infty \tag{6}$$

Second, the remainder of the energy flux, which is largely wasted, since it serves only to heat the environment, can be expresses as the kinetic energy flux relative to the environment.

Mathematically, this "leaving loss" or power which is lost to the environment may be expressed as,

$$P_L = (\rho/2)AV_{j\infty}(V_{j\infty} - U_\infty)^2 \qquad (7)$$

and as can be observed by a simple addition of Equations 6 and 7, the jet power (P_j), is the sum of the thrust power and the leaving loss, or;

$$P_j = P_F + P_L \qquad (8)$$

The percentage of the jet power (P_j), which is converted to thrust power is commonly called the propulsive efficiency. Thus

$$\eta_{\text{free jet}} = P_F/P_j \qquad (9)$$

b) <u>Ejector</u>

The influence of ejector thrust augmentation upon the propulsive efficiency is most easily described in terms of the thrust augmentation (ϕ), which is defined as,

$$\phi = \frac{\text{EJECTOR THRUST}}{\text{REFERENCE JET THRUST}} = F_{ej}/F_{ref} \qquad (10)$$

where the reference jet is a free jet utilizing the same jet power and flow rate as that of the ejector's primary jet.

The exact definition of the thrust augmentation will be discussed in more detail in the following section, but for purposes of the present discussion it is sufficient to observe that the propulsive efficiency of an ejector can be expressed in terms of the propulsive efficiency of its reference jet, with the use of the thrust augmentation as defined above. Since as indicated by Equations 6,

$$P_{F,ref} = F_{ref} \times U_\infty \qquad (11)$$

and using Equation 10,

$$P_{F,ej} = F_{ej} \times U_\infty = \phi F_{ref} \times U_\infty = \phi P_{F,ref} \qquad (12)$$

and therefore since $P_{j,ej} = P_{j,ref} = P_j$, by definition of the properties of the reference jet, the propulsive efficiency of an ejector is,

$$\eta_{ej} = P_{F,ej}/P_j = \phi P_{F,ref}/P_j = \phi\eta_{ref} \qquad (13)$$

where ϕ is the thrust augmentation ratio of the ejector, which is always greater than 1.0, for useful ejector applications.

Thus the thrust augmentation (ϕ) is the ratio of ejector thrust to reference jet thrust, and in addition, ϕ represents the ratio of the propulsive efficiency of the ejector to that of the reference jet.

As indicated by Equation 12, the useful power (P_F) of the system is increased by the factor ϕ, when an ejector is utilized. This increase of useful power is achieved at the expense of the otherwise wasted power (P_L).

An ejector therefore may be considered as a means for conversion of a portion of the (otherwise wasted) power (P_L), into useful propulsive power (P_F).

c) **Thrust Augmentation Ratio**

To describe the thrust augmentation ratio ϕ of an ejector in a meaningful manner, it is essential that the thrust of the ejector (F_{ej}) be compared to the thrust of a free (unaugmented) reference jet (F_{ref}), driven by the same power supply, operating at the same condition as that which energizes the primary jet of the ejector.

Thus if,

$$\phi = F_{ej}/F_{ref} \tag{14}$$

the reference jet must be operating with the same jet power and flow rate as that of the ejector's primary jet.

Since the primary jet of the ejector is not necessarily exhausting to ambient pressure, the primary jet velocity V_p may differ from the jet velocity $V_{j\infty}$ of the reference jet. Despite this however, the ejector's primary nozzle exit area (a), can be designed to assure that the flow rate through this nozzle Q_p is equal to the flow rate of the reference jet Q_{ref}. In other words,

$$Q_p = a \times V_p = Q_{ref} = a_\infty \times V_{j\infty} = Q \tag{15}$$

where

a_∞ = area of a free (reference) jet having the same flow rate and jet power as that of the ejector's primary jet.

Since the jet power of the ejector's primary jet $P_{j,ej}$ must be equal to the jet power of the reference jet $P_{j,ref}$, then for an incompressible fluid,

$$P_{j,ej} = Q_p(\Delta P)_p = P_{j,ref} = Q_{ref}(\Delta P)_{ref} \tag{16}$$

where

ΔP_{ref} = the velocity head pressure rise required from the pump to produce the jet velocity $V_{j\infty}$.

Obviously, as indicated by Equation 16, the value of $(\Delta P)_p$ must be equal to the value of $(\Delta P)_{ref}$ if the equality of jet power and flow rate is to be maintained.

Bernoulli's Equation for an incompressible flow through a pump provides the relationship,

$$(\Delta P) = (\rho/2)(V_{j\infty}^2 - U_\infty^2) \tag{17}$$

whether the pump is exhausting to ambient pressure or to some other pressure since the exhaust pressure influences the jet velocity but not the power (nor ΔP), required to attain that velocity.

Since it has been shown that

$$(\Delta P)_{ej} = (\Delta P)_{ref} \tag{18}$$

and in view of Equation 17, the velocity of the ejector's primary jet ($V_{j\infty,ej}$), if expanded to ambient pressure, will be equal to the velocity of the reference jet $V_{j\infty}$. Thus the thrust of the reference jet can be expressed in terms of the parameters which are measured in an ejector experiment, since,

$$F_{ref} = \rho a_{\infty} V_{j\infty}(V_{j\infty} - U_{\infty}) = \rho Q(V_{j\infty} - U_{\infty}) \tag{19}$$

where, in view of Equation 17,

$$V_{j\infty}^2 = 2(\Delta P)/\rho + U_{\infty}^2 \tag{20}$$

Thus $V_{j\infty}$ can be evaluated in terms of (ΔP), ρ, and U_{∞}, and from this, the thrust of the reference jet can be determined from Equations 19 and 20.

Since the ejector discharges to ambient pressure, its net thrust F_{ej} can be expressed as the net change of momentum flux of the total system fluid.

$$F_{ej} = \rho A_{ej.exit} U_{ej.exit}(U_{ej.exit} - U_{\infty}) \tag{21}$$

and the thrust augmentation can then be evaluated as the ratio of ejector net thrust to the net thrust of the reference jet, as indicated by Equation 14.

Evaluation of the ejector's exit velocity in terms of the velocity of the reference jet, to evaluate the thrust augmentation is described in a later section of this document.

d) Total Power

It is important to note at this point, that the total power required to drive the jet exceeds the jet power P_j, due to the duct losses upstream of the pump, nozzle losses, and other energy losses of the system.

These losses can be evaluated for any given installation, and combined into a head factor H. The total power required P_T can then be determined as,

$$P_T = P_j + QH \tag{22}$$

Skin friction and incomplete mixing are responsible for the deviation of experiment from the theory (which neglects skin friction and assumes complete mixing). These influences can be evaluated experimentally and the data can then be utilized to formulate an empirical correction to the idealized theory.

e) **Relative Power**

While the thrust augmentation of an ejector is a very useful parameter for evaluation of the performance of the ejector, it is essential, from the point of view of the system design, to evaluate the ratio of the jet power of an augmented thrusting system to the jet power of an unaugmented system, when both systems are producing the same net thrust (F).

If the net thrust of an ejector (F_{ej}), is equal to the net thrust of a free jet (F_o),

$$F_{ej} = \rho a_\infty V_{j\infty}(V_{j\infty} - U_\infty)\phi = F_o = \rho A_o V_{j,o}(V_{j,o} - U_\infty) \tag{23}$$

or in an incompressible fluid, using Equation 3,

$$R_P = P_{j,ej}/P_{j,o} = Q_{p,ej}(\Delta P)_{p,ej}/Q_o(\Delta P)_o = \{(V_{j\infty} + U_\infty)/(V_{j,o} + U_\infty)\}(1/\phi) \tag{24}$$

Since the thrust and jet power are not uniquely related, it is essential to impose one other condition upon the system, in order to obtain a comparison. Two alternative conditions will be considered.

First, assume that the free jet and the augmented jet utilize equal energized flow rates, or that,

$$Q_{p,ej} = Q_o \tag{25}$$

Under this condition, Equation 24 can be written with the aid of Equation 3 as,

$$(\Delta P)_{p,ej}/(\Delta P)_o = (V^2_{j\infty} - U^2_\infty)/(V^2_{j,o} - U^2_\infty) = \{(V_{j\infty} + U_\infty)/(V_{j,o} + U_\infty)\}(1/\phi) \tag{26}$$

or

$$\phi(V_{j\infty} + U_\infty) = (V_{j,o} + U_\infty) + 2(\phi - 1)U_\infty \tag{27}$$

and therefore, eliminating $V_{j\infty}$ between Equations 24 and 27 results in,

$$R_P = P_{j,ej}/P_{j,o} = (1/\phi)^2 \{1 + 2(U_\infty/V_{j,o})(\phi-1)/(1 + (U_\infty/V_{j,o}))\} \tag{28}$$

Thus, as shown by Equation 28, the power ratio (R_P), is a function of the thrust augmentation (ϕ) of the ejector and the ratio of the velocity of the vehicle (U_∞), to the jet velocity of the free jet ($V_{j,o}$).

A plot of this relationship is presented on Figure 9, to illustrate the large advantage in power requirements available as a result of thrust augmentation.

Further discussion of this power ratio (R_P), will be presented in the following section of this document, where a method for evaluation of the parameters (ϕ and $U_\infty/V_{j,o}$), in terms of the ejector's geometry, operational condition, and power supply characteristics, is described.

Second, assume that the free jet and the augmented jet have equal jet velocity pressure increments, or equivalently $V_{j,o} = V_{j\infty}$.

Under this condition Equation 24 reduces to the expression

$$R_P = P_{j,ej}/P_{j,o} = (1/\phi) \qquad (29)$$

This comparison, in which the jet velocity of the free jet ($V_{j,o}$) is assumed to be equal to the jet velocity of the ejector's primary jet ($V_{j\infty}$), when discharged to ambient pressure, results in a dependence of the power ratio (R_P), upon the thrust augmentation of the ejector (ϕ). Under this condition the pump flow rate for the ejector is smaller than that for the free jet, by the factor $1/\phi$, as indicated by Equation 24. This relationship is also illustrated on Figure 9, where it is shown to be identical to the previous case, when the ratio $U_\infty/V_{j,o}$ is equal to 1.0.

Although no valid reason for utilization of either of the above two types of comparison, can be made at this stage of the investigation, it is apparent that the final choice will depend upon the results of a detailed system analysis, where the weights of the installed systems, the fuel consumption, and the system size are determined for a given system mission.

Further elaboration of these concepts will be discussed in the following section of this document, where a method for evaluation of the thrust augmentation of the ejector, in terms of the ejector's geometry, operational condition and power supply characteristics, is described.

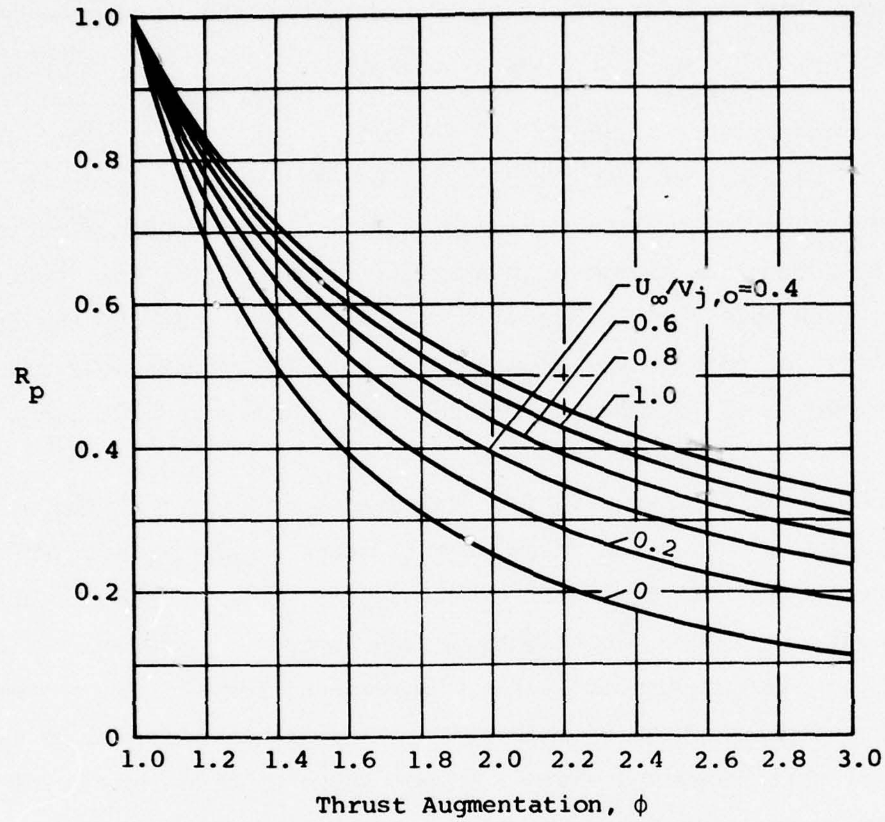


Figure 9. INFLUENCE OF VELOCITY RATIO ON RELATIVE POWER

SECTION III

THRUST AUGMENTING EJECTOR IN MOTION IN AN INCOMPRESSIBLE FLUID

Many modern naval vehicles utilize boundary layer control to reduce the skin friction drag. These fluids can be utilized in the propulsion system to provide additional performance gains.

For this reason, the analysis of the flow through an ejector will be carried out under the simplified assumption that the pump and ejector flows originate at regions where the velocities are different from the free-stream velocity. Evaluation of these velocities from experimental data involves an evaluation of the mean entrance velocity and an averaging of the flow-rate weighted stagnation pressure distribution at the inlet sections, in order to compensate for the pressure difference between the inlet section and the free-stream.

Consider a translating ejector, located with respect to the vehicle's boundary layer, such that its pump inlet velocity is U_p, and its ejector inlet velocity is U_e. Thus the classical free-stream case can be considered as a special case where $U_p = U_e = U_\infty$.

In order to provide a uniformity of notation, and for the purpose of later comparison with ejector performance, the thrust and propulsive efficiency of a free jet translating at a velocity U_∞, will be derived in a convenient form as follows.

When the pump inlet is in the boundary layer (or wake), where the average velocity is U_p, the net thrust F will be,

$$F_{ref} = \rho a_\infty V_{j\infty}(V_{j\infty} - U_p) \tag{30}$$

The jet power required to energize the jet when the pump inlet is in the boundary layer is

$$P_{j,ref} = (\rho/2) a_\infty V_{j\infty}(V_{j\infty}^2 - U_p^2) \tag{31}$$

and the propulsive efficiency under this condition is,

$$\eta_{ref} = F_{ref}(U_\infty/P_j) = \frac{2(V_{j\infty} - U_p)U_\infty}{V_{j\infty}^2 - U_p^2} = \frac{2U_\infty/V_{j\infty}}{1 + U_p/V_{j\infty}} \tag{32}$$

where

ρ = mass density of the fluid
a_∞ = area of the reference jet nozzle
$V_{j\infty}$ = velocity of the jet fluid when expanded to ambient pressure
U_p = average velocity of the pump inlet flow

These quantities can be expressed in a more convenient form for later use in comparison with ejector performance by substitution of the parameters,

$$q = (\rho/2)U^2 \tag{33}$$

and (ΔP), which is defined by Bernoulli's relationship through the pump as,

$$p_\infty + (\rho/2)U_p^2 + \Delta P = p_\infty + (\rho/2)V_{j\infty}^2 \tag{34}$$

Thus the velocity ratios appearing in the expression for propulsive efficiency can be expressed in terms of q and (ΔP) as follows,

$$(U_p/V_{j\infty})^2 = q_p/(\Delta P + q_p) \tag{35}$$

and

$$(U_\infty/V_{j\infty})^2 = q_\infty/(\Delta P + q_p) \tag{36}$$

substitution of 35 and 36 into the expressions for thrust, jet power and propulsive efficiency results in the following;

$$F_{ref} = \rho a_\infty V_{j\infty}^2 \{ 1 - \sqrt{q_p/(\Delta P + q_p)}\} \tag{37}$$

$$P_{j,ref} = (\rho/2)a_\infty V_{j\infty}^3 \{ 1 - q_p/(\Delta P + q_p)\} \tag{38}$$

and

$$\eta_{ref} = 2 \sqrt{q_\infty/(\Delta P)} \{ \sqrt{1 + q_p/(\Delta P)} - \sqrt{q_p/(\Delta P)}\} \tag{39}$$

Equations 37, 38, and 39 are identical to the expressions in Equations 30, 31, and 32, but the use of the parameters q and (ΔP), provides more insight into the dependence of ejector performance on the jet power than can be obtained by use of the velocities, particularly in view of the fact that the jet power is related to the velocity head pressure rise (ΔP), in a simple manner as indicated by Equation 3.

The analysis of the flow through an ejector can now be performed under the same conditions whereby the velocities at the inlets of the pump and ejector are assumed to have a magnitude lower than that of the free stream velocity U_∞.

1. EJECTOR FLOW ANALYSIS

Consider a conventional ejector, configured as illustrated schematically on Figure 10.

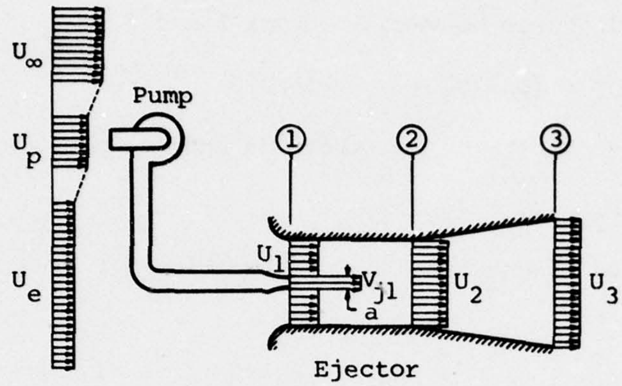


Figure 10. SCHEMATIC ILLUSTRATION OF TYPICAL UNDERWATER EJECTOR

If the primary nozzle exit is located within the ejector's shroud, as illustrated, the pressure at the nozzle exit will be less than ambient. In that case, the primary jet of the ejector is not identical to the reference jet used to define the thrust augmentation.

If the reference jet exit area is a_∞, then to maintain the equality of flow rate between the primary jet of the ejector and the reference (free) jet, the ejector's primary jet exit area must be,

$$a = a_\infty (V_{j_\infty}/V_{j1}) \tag{40}$$

where

a = primary jet exit area

V_{j1} = velocity of the fluid effluent from the primary jet at Station 1

With this understanding, and assuming that between Stations 1 and 2 the

Mixing of primary and induced flows occurs
Ejector duct has a constant cross-section
Skin friction is negligible

then the momentum balance between Stations 1 and 2 can be written as

$$p_1 A_2 + \rho A_1 U_1^2 + \rho a V_{j1}^2 = p_2 A_2 + \rho A_2 U_2^2 \tag{41}$$

since

$$A_2 = A_1 + a \tag{42}$$

The pressure terms in Equation 41 can be expressed in terms of the velocities with the aid of Bernoulli's Equation as follows;

At the entrance to the ejector,

$$p_\infty + (\rho/2)U_e^2 = p_1 + (\rho/2)U_1^2 \tag{43}$$

and within the diffuser between Stations 2 and 3,

$$p_\infty + (\rho/2)U_3^2 = p_2 + (\rho/2)U_2^2 \tag{44}$$

The difference between Equations 43 and 44, provides the result,

$$P_2 - P_1 = (\rho/2)\ (U_3^2 - U_2^2 + U_1^2 - U_e^2) \tag{45}$$

Substitution of Equation 45 into Equation 41, results in the expression,

$$\lambda_3^2 + \lambda_2^2 - \{(\alpha - 2)/\alpha\}(\lambda_1)^2 - \lambda_e^2 - 2/\alpha = 0 \tag{46}$$

where

$$\lambda_n = U_n/V_{j1} \tag{47}$$

and

$$\alpha = A_2/a \tag{48}$$

Expression of λ_1 and λ_2 in terms of λ_3 and the ejector's geometry can be accomplished with the aid of the law of mass flow conservation, which can be expressed mathematically as,

$$A_1U_1 + aV_{j1} = A_2U_2 = A_3U_3 \tag{49}$$

or

$$(\alpha - 1)\lambda_1 + 1 = \alpha\lambda_2 = \alpha\delta\lambda_3 \tag{50}$$

where

$\delta = A_3/A_2$ = the diffuser area ratio

Elimination of λ_2 from Equation 46, with the aid of Equation 50, results in an expression for λ_3 in terms of λ_1 and the geometric and operational parameters which are given. This expression is,

$$\lambda_3^2 = \lambda_e^2 + (1/\alpha)^2\ \{2\alpha(1 - \lambda_1) - (1 - \lambda_1)^2\} \tag{51}$$

This form of the relationship among the dimensionless velocities is very useful, as will be shown, for use in consideration of the cavitation limitation.

It is convenient at this point to express λ_e in terms of the dynamic pressure and ΔP, in a manner corresponding to that which was utilized in Equations 37, 38, and 39.

To accomplish this transformation, consider Bernoulli's Equation; at the entrance to the ejector, which is

$$p_\infty + (\rho/2)U_e^2 = p_1 + (\rho/2)U_1^2 \tag{52}$$

and through the pump, Bernoulli's Equation is,

$$p_\infty + (\rho/2)U_p^2 + (\Delta P) = p_1 + (\rho/2)V_{j1}^2 \tag{53}$$

solving for the term V_{j1}, results in the expression,

$$(\rho/2)V_{j1}^2 = \{(\Delta P) - q_e + q_p\} /(1 - \lambda_1^2) \tag{54}$$

therefore,

$$\lambda_e^2 = (U_e/V_{j1})^2 = q_e(1 - \lambda_1^2)/(\Delta P)_{eff} \tag{55}$$

where

$$(\Delta P)_{eff} = (\Delta P) - q_e + q_p \tag{56}$$

Substitution of Equations 50 and 55 into Equation 51, provides the desired result, which is,

$$A\lambda_3^2 + B\lambda_3 + C = 0 \tag{57}$$

where

$$A = (\alpha - 1)^2 + \delta^2 + \alpha^2\delta^2\{q_e/(\Delta P)_{eff}\} \tag{58}$$

$$B = 2\delta(\alpha - 2) - 2\alpha\delta \{q_e/(\Delta P)_{eff}\} \tag{59}$$

$$C = -(2\alpha - 3) - \{(\alpha - 1)^2 - 1\} \{q_e/(\Delta P)_{eff}\} \tag{60}$$

and the solution of the quadratic equation is,

$$\lambda_3 = \{-B + \sqrt{B^2 - 4AC}\} /2A \tag{61}$$

The other solution of the quadratic equation is discarded since it produces negative values for the velocity.

Thus for any given ejector geometry, q_e, q_p and (ΔP), the value of λ_3 can be determined.

Having the value of λ_3, the thrust augmentation and propulsive efficiency of the ejector system can be evaluated as follows.

2. THRUST AUGMENTATION

The general case under consideration, in which the inlet velocities at the pump and the ejector are different from each other and different from the free stream velocity, requires special consideration in the determination of the ejector's net thrust.

Since $U_e \neq U_p$, the momentum decrement due to the ingestion of fluid from the environment (ram drag), must be considered separately for the primary and induced flows.

In terms of the notation used to describe the flow through the ejector, the flow rate of the induced flow Q_i, can be expressed as,

$$Q_i = A_3U_3 - aV_{j1} \tag{62}$$

and the flow rate of the primary flow Q_p can be expressed as,

$$Q_p = aV_{j1} \tag{63}$$

The ejector's net thrust is then,

$$F_{ej} = \rho Q_i(U_3 - U_e) + \rho Q_p(U_3 - U_p) \tag{64}$$

and the net thrust of the reference jet is

$$F_{ref} = \rho Q_p(V_{j\infty} - U_p) \tag{65}$$

Thus in general, the thrust augmentation in terms of the notation used here is,

$$\phi = F_{ej}/F_{ref} = \frac{(\alpha\delta\lambda_3 - 1)(\lambda_3 - \lambda_e) + (\lambda_3 - \lambda_p)}{V_{j\infty}/V_{j1} - \lambda_p} \tag{66}$$

where, in view of Equations 34 and 54,

$$(V_{j\infty}/V_{j1})^2 = (q_p + \Delta P)(1 - \lambda_1^2)/(\Delta P)_{eff} \tag{67}$$

and expressing λ_e and λ_p in terms of $q_e/(\Delta P)_{eff}$ and $q_p/(\Delta P)_{eff}$ as in Equation 55, the thrust augmentation can be written in the form,

$$\phi = \frac{\alpha\delta\lambda_3\ \{\lambda_3/\sqrt{1 - \lambda_1^2} - \sqrt{q_e/(\Delta P)_{eff}}\} - \{\sqrt{q_p/(\Delta P)}_{eff}(1 - \sqrt{q_e/q_p})\}}{\sqrt{\{1 + q_p/(\Delta P)\}\{(\Delta P)/(\Delta P)_{eff}\}} - \sqrt{q_p/(\Delta P)_{eff}}} \tag{68}$$

Using Equation 61 to evaluate λ_3, then using Equation 50 to determine the corresponding value of λ_1, the thrust augmentation can be determined for any given set of geometrical and operational characteristics with the aid of Equation 68.

Another, more useful form of the expression for thrust augmentation can be derived with the aid of Equations 50, 51, and 55.

Expressing the term $\alpha\delta\lambda_3$, in terms of α and λ_1 only, as given by Equation 50, provides the relationship

$$\alpha\delta\lambda_3 = 1 + (\alpha - 1)\lambda_1 \tag{69}$$

and using Equations 51 and 55, the term $\lambda_3/\sqrt{1 - \lambda_1^2}$, can be written as

$$\lambda_3/\sqrt{1 - \lambda_1^2} = \sqrt{q_e/(\Delta P)_{eff} + (1/\alpha)^2 \{1 + 2(\alpha-1)/(1+\lambda_1)\}} \tag{70}$$

Substituting Equations 69 and 70 into Equation 68 provides the expression for thrust augmentation in terms of λ_1, and α, in a form which does not involve λ_3 nor δ. This expression is,

$$\phi = \frac{\{1+(\alpha-1)\lambda_1\}\left\{\sqrt{\dfrac{q_e}{(\Delta P)_{eff}} + (1/\alpha)^2\left[1+\dfrac{2(\alpha-1)}{1+\lambda_1}\right]} - \sqrt{\dfrac{q_e}{(\Delta P)_{eff}}}\right\} - \sqrt{\dfrac{q_p}{(\Delta P)_{eff}}}\left(1-\sqrt{\dfrac{q_e}{q_p}}\right)}{\sqrt{\{1 + q_p/(\Delta P)\}\ (\Delta P)/(\Delta P)_{eff}} - \sqrt{q_p/(\Delta P)_{eff}}} \tag{71}$$

This equation for thrust augmentation in terms of λ_1, as the only independent variable, will be useful in the calculation of ϕ at the limit of cavitation.

3. PROPULSIVE EFFICIENCY

The thrust augmentation is defined, as described previously (Section II.3) as the net thrust of an ejector, compared to the net thrust of a free (reference) jet whose mass flow and jet power are equal to those of the ejector's primary jet.

From the definition of propulsive efficiency, therefore, it is evident that the propulsive efficiency of the ejector system is related to the propulsive efficiency of the reference jet as,

$$\eta_{ej} = U_\infty F_{ej}/P_{j,ej} = U_\infty \phi F_{ref}/P_{j,ref} = \phi\eta_{ref} \tag{72}$$

Thus the propulsive efficiency of the ejector system can be determined by the product of Equation 39 times Equation 68 or 71.

4. RELATIVE POWER (R_p)

Assuming the pump inlet is located in the flow where the average velocity is U_p, and that the ejector's net thrust (F_{ej}) is equal to the net thrust of a free (unaugmented) jet (F_o) then,

$$F_{ej} = \rho Q_{p,ej}(V_{j\infty} - U_p)\phi = F_o = \rho Q_o(V_{j,o} - U_p) \tag{73}$$

Under these conditions the jet power required to drive the ejector ($P_{j,ej}$) can be expressed in terms of the jet power required to drive the unaugmented jet ($P_{j,o}$) as,

$$R_p = P_{j,ej}/P_{j,o} = (\rho/2)Q_{p,ej}(V_{j\infty}^2 - U_p^2)/\{(\rho/2)Q_o(V_{j,o}^2 - U_p^2)\} \tag{74}$$

Combination of Equations 73 and 74, results in the expression,

$$R_p = (V_{j\infty} + U_p)/\ \phi(V_{j,o} + U_p) \tag{75}$$

A combination of Equations 74 and 75 simplifies to the expression,

$$(Q_{p,ej}/Q_o)\ \{(V_{j\infty} - U_p)/(V_{j,o} - U_p)\} = 1/\phi \tag{76}$$

Since this equation involves two independent variables, it is necessary to impose one additional constraint in order to obtain a unique solution.

This additional condition can be any of several different constraints which determine the relationship among the flow rates, jet velocities or areas of the free jet and the ejector.

Consider first the condition where,

$$Q_{p,ej} = Q_o \tag{77}$$

Under this condition Equation 76 becomes,

$$V_{j,o} + U_p = \phi(V_{j\infty} + U_p) - 2U_p(\phi - 1) \tag{78}$$

Eliminating $V_{j,o}$ between Equations 75 and 78 reduces to the expression,

$$R_p = (V_{j\infty} + U_p)/\{\phi\ \{\phi(V_{j\infty} + U_p) - 2U_p(\phi - 1)\}\} \tag{79}$$

which can be expressed in terms of η_{ej} with the aid of Equations 32 and 72 as follows,

$$R_p = 1/\{\phi^2 - (\phi - 1)\eta_{ej}(U_p/U_\infty)\} \tag{80}$$

A second condition whereby the jet velocity of the ejector's primary jet, expanded to ambient pressure ($V_{j\infty}$), is assumed equal to the jet velocity of the unaugmented jet ($V_{j,o}$), may be imposed upon the relationship (Equation 76). Thus since,

$$V_{j\infty} = V_{j,o} \tag{81}$$

it follows from Equation 75 that

$$R_P = 1/\phi \tag{82}$$

and as can be observed from Equation 73, the flow rate through the ejector's pump is smaller than that through the free jet by the factor R_P.

The two conditions represented by Equations 80 and 82 result in different values for the power ratio (R_P).

Under either assumption however, the use of thrust augmentation proves to provide a large advantage in power requirements.

It is important to note here that the reduced power required by augmented systems is reflected in savings of the system weight, size and energy consumption rate, as well as in the external drag in some applications.

Detailed considerations of these savings must be relegated to the system design where the importance of the various elements of the system will depend upon the operational conditions, vehicle size and weight constraints and upon the mission. These considerations are beyond the scope of this document and, for the present, the information will be discussed in the very general form above, and the evaluation of the relative power for some special types of ejector installations will be discussed in the following sections.

5. OPTIMIZATION OF EJECTOR PERFORMANCE

As indicated by the equations for thrust augmentation and propulsive efficiency, these quantities are functions of a great number of parameters.

These parameters include;

Ejector geometry (inlet area ratio (α), and diffuser area ratio (δ))

Operational conditions (velocity (U_∞), depth, U_e, and U_p)

Properties of the injected fluid (flow rate (Q), ΔP, and density (ρ))

With modern computer techniques, the determination of the performance of an ejector under any desired values of these parameters, can easily be evaluated, once the program exists. However, for purposes of this document, the performance achievable by an ejector can be reduced to a set of charts, for some special cases. Other more complex applications will require the use of computer solutions.

The optimization of ejector geometry and power supply characteristics can be somewhat simplified, by consideration of the conditions at which the thrust augmentation reaches its maximum value for any given operational specifications. This may be accomplished as follows.

Since the expression for thrust augmentation (Equation 71), is a function of λ_1 only (for given q_e, q_p, ΔP and α), it is of interest to examine the influence of p_1, the smallest pressure within the ejector duct, upon the thrust augmentation, with the aid of Equation 71.

If then, λ_1 is expressed in terms of the pressure p_1, and the known quantities, it would be possible to determine ϕ as a function of p_1.

Recognizing the fact that,

$$\lambda_1^2 = (U_1/V_{j1})^2 = \frac{p_\infty + q_e - p_1}{p_\infty + q_p + \Delta P - p_1} = \frac{1 - p_1/(p_\infty + q_e)}{P_{op}/(p_\infty + q_e) - P_1/(p_\infty + q_e)} \qquad (83)$$

where

$$P_{op} = p_\infty + q_p + \Delta P = \text{Plenum or stagnation pressure of primary jet}$$

Thus the desired relationship between λ_1 and p_1, is described by Equation 83, and the thrust augmentation can be evaluated in terms of p_1 and the known parameters of the problem, through Equations 71 and 83.

The dependence of thrust augmentation over the entire range of values of $p_1/(p_\infty + q_\infty)$, with $q_\infty/\Delta P$ fixed at a value of 0.05, and with $\Delta P/p_\infty$ as a parameter, is presented on Figure 11, for the condition where $U_e = U_p = U_\infty$.

As illustrated on Figure 11, the thrust augmentation increases monotonically with decreasing values of p_1, when the free stream condition and (ΔP) are fixed. Although this is shown for a special case only, it is true for all pump and ejector inlet conditions, and for all realistic ejector geometries, speeds, depths, etc. The rigorous proof of this fact is left to the reader, in order to avoid unnecessary mathematical complexity at this point of the discussion.

The variation of p_1 illustrated on Figure 11, requires changes in the diffuser area ratio δ, as illustrated. This change in diffuser area ratio also is accompanied by a change of the ejector inlet, to provide optimal inlet design. Although not relevant to the present discussion, the use of small values of δ, implies (at some point described on Figure 11) that the inlet would be decelerating, (or diffusing) as is sometimes utilized in pump jet design.

As indicated, the use of these small diffuser area ratios (sometimes less than 1.0) results in ejector performance far from optimal. Thus it is evident that to achieve maximum ejector thrust augmentation, the design must be such that the flow at Station 1 is at a pressure close to zero.

Since the pressure at which sea water cavitates is always less than 3% of surface pressure, (p_{cav} is less than 20 mm Hg for sea water at the temperatures encountered) and in view of the mathematical simplification, the optimal ejector performance is determined under the condition that $P_1 = 0$.

As can be observed from Figure 11, this results in a very small advantage over the calculation using actual cavitation pressure at Station 1, since the slope of ϕ vs $p_1/(p_\infty + q_\infty)$ is a minimum at that point.

The evaluation of the thrust augmentation when $p_1 = 0$, provides a value close to the maximum achievable under any given set of operational and geometric ejector parameters, and permits the presentation of the data with a minimal of complexity, as a result of the reduction of the number of independent parameters.

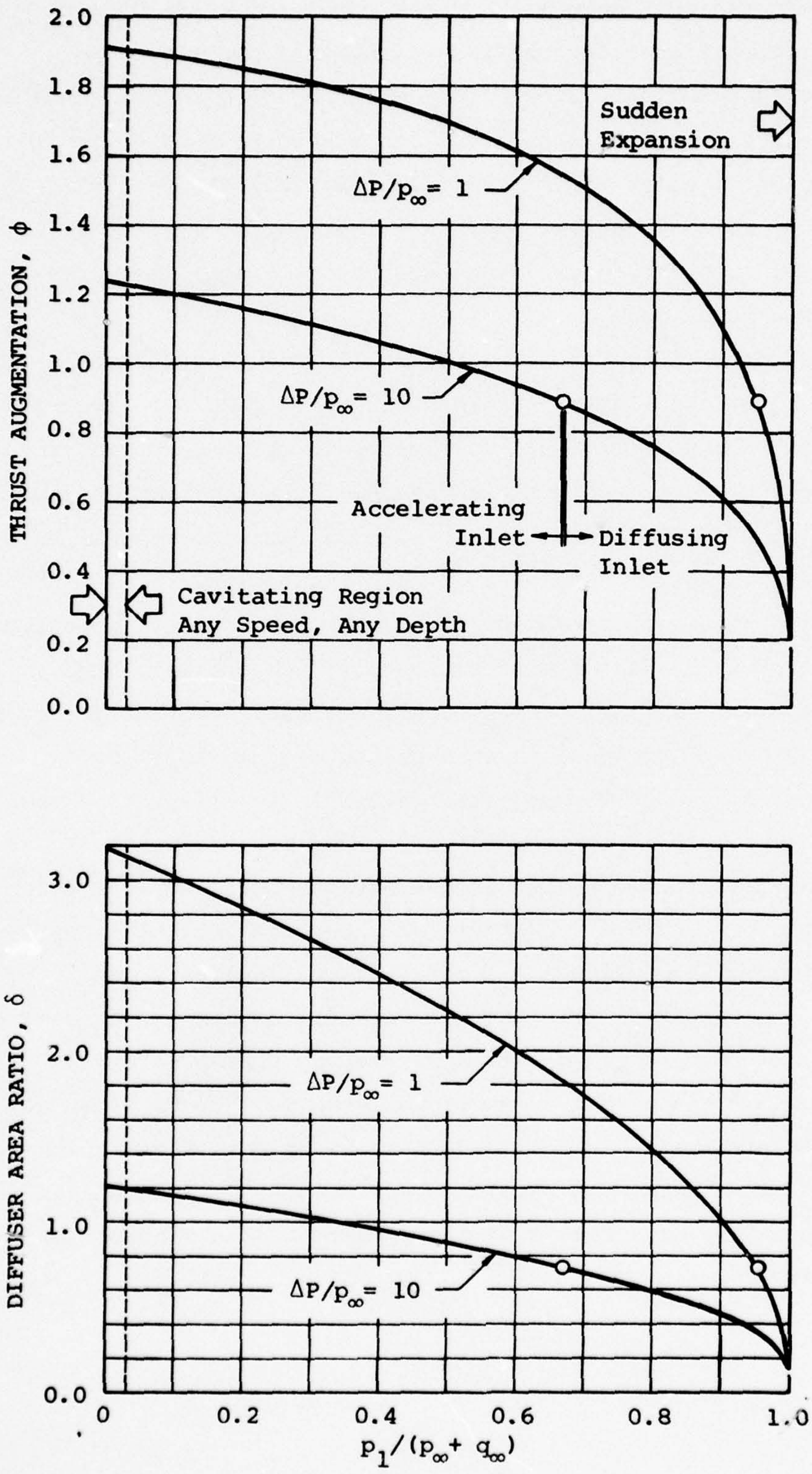


Figure 11. EJECTOR PERFORMANCE OPTIMIZATION

a) Non-cavitating Ejector Performance

Considering the ejector geometry at which $p_1 = 0$ as a critical design, the value of λ_1 (when p_1=0) can be designated $\lambda_{1,c}$ whose value can be established from Equation 83 as,

$$\lambda^2_{1,c} = \frac{p_\infty + q_e}{p_\infty + q_p + (\Delta P)} = P_{oe}/P_{op} = \frac{1 + (q_e/\Delta P)(\Delta P/p_\infty)}{1 + (q_p/\Delta P)(\Delta P/p_\infty) + (\Delta P/p_\infty)} \tag{84}$$

and inserting this value of $\lambda_{1,c}$ into Equation 71, provides the maximum, achievable value of thrust augmentation, at any given set of ejector geometric and operational conditions.

Under this condition the value of $\lambda_{3,c}$ can be evaluated with the aid of Equation 70, and using that value of $\lambda_{3,c}$ the diffuser area ratio (δ_c), can be determined from Equation 69. If the value of δ_c, obtained in this manner is excessively large, (too large for the geometry of the vehicle, or beyond the value which can be utilized without separation), smaller values of δ can be used. This however, will require use of Equations 58 to 61, for the solution for λ_3, and Equation 50 for λ_1. The thrust augmentation can then be evaluated with Equation 68 or 71.

Ejector performance calculated as above represents a comparison of ejector thrust compared to the thrust of a free (reference) jet. When however, the reference jet and/or the ejector are injesting fluid having a velocity less than free stream velocity, the effluent flow may also have a velocity lower than free stream. The thrust under these conditions is less than the drag represented by the momentum decrement of the injested fluid. Ejectors like free jets or propellers, can produce thrust under this condition but the magnitude of the thrust will be smaller than the vehicle drag. This operating regime represents a useful thruster for control purposes, but cannot be utilized as the sole propulsive device for the vehicle.

To assure the production of a net thrust at least equal to the drag of the vehicle, the value of U_3 must be greater than U_∞. The limiting condition where $U_3 = U_\infty$ can be determined from an evaluation of $(\lambda_3 - \lambda_\infty)$, where λ_∞ can be calculated from Equation 55 by replacing subscript "e" with "∞", and λ_3 can be calculated from Equation 61. Thus the difference $(\lambda_3 - \lambda_\infty)$ can be determined, and if the difference is negative, the thrust must be less than the drag.

Self propelling thrusters must have positive value of $(\lambda_3 - \lambda_\infty)$.

Using the criterion that $P_1 = 0$, some special cases of ejector performance have been analyzed, and are discussed in the following section.

b) <u>Thrust/Drag Considerations</u>

When a finite ideal ejector is operating in the free-stream, the ejector's exit velocity (U_3) will be greater than the velocity of translation (U_∞), providing the jet velocity pressure rise of the primary jet (ΔP), is greater than zero.

Under free-stream conditions, the thrust must be compared to the total drag of the vehicle, over a range of speeds, to establish the speed at which equilibrium is reached.

If however, the ejector inlet is located in the vehicle's boundary layer or wake, the equilibrium point or speed at which the thrust equals the drag, can be determined from a knowledge of the average velocity at the ejector's inlet.

In general, the thrusting ejector must make up the momentum deficiency resulting from the vehicle drag, in addition to the momentum deficiency due to the injestion of the entrained flow (ram drag).

Assurance that the thrust is greater than the combined effect of skin friction and ram drag, can be obtained if the ejector is designed so that its exit velocity (U_3) is at least equal to the vehicle velocity (U_∞).

This can be assured if the inlet area ratio (α), is smaller than the value at which U_3 equals U_∞.

This limiting value of inlet area ratio (α_{max}), can be evaluated by assuming that $\lambda_3 = \lambda_\infty$, and rearranging Equation 70 to solve for α. The resulting expression for α_{max} is as follows,

$$\alpha_{max} = \{(\Delta P)_{eff}/\{(1+\lambda_1)(q_\infty - q_e)\}\}\{1 + \sqrt{1 - (1-\lambda_1^2)(q_\infty - q_e)/(\Delta P)_{eff}}\} \tag{85}$$

Values of α in excess of α_{max} will produce a positive thrust but this thrust will be smaller than the drag represented by the momentum deficiency of the injested fluid. Such designs produce very high thrust augmentation and propulsive efficiencies, but are only useful for applications where the thrust force is not the sole means of propulsion. For example, such devices may be utilized for control purposes.

c) Drag/skin friction considerations

To avoid excessive drag, a moving ejector must be designed with its inlet similar to the leading edge of a subcavitating hydrofoil or a subsonic airfoil. In addition, the cavitation phenomenon restricts the diffuser area ratio (δ), to small values compared to those utilized in ejectors operating in air. In fact as the speed is increased the maximum diffuser area ratio which can be utilized, decreases, and the cross-section of the ejector becomes similar to that of a subcavitating hydrofoil or subsonic airfoil.

Although the performance calculations presented in this document relate to a conventional ejector (with solid diffusers), the eventual goal of this program involves the use of jet diffuser ejectors, in view of the large size and weight advantage of this concept. Configuration and performance details of jet diffuser ejectors are presented in Reference 1,

The performance analysis presented in this document was not based upon the jet diffuser ejector primarily due to the fact that methods for analysis of this type of ejectors in motion do not presently exist. However, the considerations of the drag will be based upon the cross-sections of the AJDE, a two-dimensional jet diffuser ejector developed for use in air, and reported in Reference 1.

For example, if a jet diffuser ejector is travelling at 24 knots, near the sea surface, its diffuser area ratio is limited to about 1.5, or

$$X_3 = 1.5\ X_2$$

With this diffuser area ratio, the ejector duct has a length or chord (c), about 1.5 times its throat width (X_2) or,

$$c = 1.5\ X_2$$

If the external surface of the ejector is parallel to the ejector axis, the thickness of the ejector duct (t) is,

$$t = (X_3 - X_2)/2 = 0.25\ X_2 \tag{86}$$

and the thickness to chord ratio is,

$$t/c = 0.25\ X_2\ /\ 1.5\ X_2 = 17\%$$

which is a typical value for a subsonic airfoil.

At 24 knots, the Reynolds number per unit length in water is

$$Re/\ell = 4 \times 10^6 \quad (ft)^{-1}$$

and of we consider a typical jet diffuser ejector of 1.0 ft. throat width, then its chord is 1.5 ft., and the Reynolds number based on its chord is

$$Re = 6 \times 10^6$$

At higher speeds, or for larger ejectors, the Reynolds number is larger, and the drag coefficient is smaller, as indicated on Figure 66 of Reference 3.

Since the ejector consists of two walls, the drag on both the internal and external surfaces is twice that of a single airfoil.

Expressing the drag coefficient (C_D) in terms of the ejector throat area,

$$C_D = (\text{Ejector drag})/(q_e A_2) = 2q_e cC_d L/q_e A_2 = 2cC_d/X_2 \tag{87}$$

where C_d is the section drag coefficient of one side of the ejector and,
L is the length of the ejector ($A_2 = LX_2$)

Figure 67 of Reference 3 gives the section drag coefficient for several NACA 6-series airfoils of 18% thickness ratio, with Reynolds number of 6×10^6, which is comparable to one side of the ejector section discussed above. This figure indicates that the section drag coefficient is in the range of

$$C_d = 0.0035 \text{ to } 0.005$$

According to these values and Equation 87, the drag coefficient is

$$C_D = 3\ C_d = 0.011 \text{ to } 0.015 \tag{88}$$

which includes both the internal and external ejector surfaces.

Since the ejector does not require any additional surface piercing strut in excess of that required for its reference jet, the above drag estimate includes all drag components resulting from the use of the ejector, and is thus realistic.

The drag coefficient based upon the ejector's throat area and the boundary layer characteristics, as defined above, can be expressed as a drag term which modifies the ideal thrust augmentation (ϕ). Thus if the drag term (ϕ_D) is defined by the relationship,

$$\phi^* = \phi - \phi_D \tag{89}$$

where ϕ^* is the overall thrust augmentation, including the drag correction term, then ϕ_D can be expressed in terms of the thrust of the reference jet (F_{ref}) as,

$$\phi_D = \frac{\text{DRAG}}{F_{ref}} = \frac{C_D\ q_e\ A_2}{F_{ref}} = \frac{C_D\ U_e^2\ A_2}{2\ a_1 V_{j1} V_{j\infty}(1 - U_p/V_{j\infty})} \tag{90}$$

and since as in Equation 67,

$$V_{j1}/V_{j\infty} = \sqrt{(\Delta P)_{eff}/((1 - \lambda_1^2)(q_p + \Delta P))} \tag{91}$$

and since,

$$(U_p/V_{j\infty}) = \sqrt{q_p/(q_p + \Delta P)} \tag{92}$$

$$(U_e/V_{j\infty}) = \sqrt{q_e/(q_p + \Delta P)} \tag{93}$$

the drag correction term (ϕ_D), can be expressed as,

$$\phi_D = (C_D\alpha/2)(q_e/\Delta P) \sqrt{((\Delta P)/(\Delta P)_{eff})(1 - \lambda_1^2)}\{\sqrt{1 + q_p/\Delta P} + \sqrt{q_p/\Delta P}\} \tag{94}$$

where the value of λ_1 can be evaluated by the general solution, using Equations 57 through 61, and Equation 50.

For a critical ejector, the value of λ_1 can be determined using Equation 84, and the drag correction term can be evaluated from Equation 94 for any given set of operational, geometric and power supply characteristics.

SECTION IV

SPECIAL CASES OF EJECTOR PERFORMANCE

1. STATIONARY EJECTOR

Many naval applications require large thrust forces at zero, or at very small velocities. In these cases $q_e = q_p = 0$, and the thrust augmentation can be determined from the general solution, as follows.

Evaluate λ_3 for any given values of the ejector geometry (α and δ), using Equations 58 to 61, under the condition that $q_e = 0$. Therefore,

$$\lambda_3 = \{-B + \sqrt{B^2 - 4AC}\}/2A \tag{95}$$

where

$$A = (\alpha - 1)^2 + \delta^2 \tag{96}$$

$$B = 2\delta(\alpha - 2) \tag{97}$$

$$C = -(2\alpha - 3) \tag{98}$$

Using this value of λ_3, the thrust augmentation (ϕ) can be determined using Equation 68, in the form where, for the stationary case $q_e = q_p = 0$,

$$\phi = \alpha\delta\lambda_3^2/\sqrt{1 - \lambda_1^2} \tag{99}$$

where λ_1 can be determined in terms of α, δ, and λ_3, using Equation 50 in the form

$$\lambda_1 = (\alpha\delta\lambda_3 - 1)/(\alpha - 1) \tag{100}$$

This general solution is valid for all values of α and δ, but an ejector operating in water cannot utilize excessively large diffuser area ratios, since at some value of δ, ($= \delta_c$) the water will reach its vapor pressure and the value of δ must be limited to the critical value as discussed previously.

This limiting value of diffuser area ratio, can be determined from Equations 84, 51, and 69, as follows.

Determine the critical value of λ_1 using Equation 84, under the condition that $q_e = q_p = 0$ which takes the form,

$$\lambda_{1,c} = \sqrt{p_\infty/(p_\infty + \Delta P)} \tag{101}$$

Using this value of $\lambda_{1,c}$, determine $\lambda_{3,c}$ with the aid of Equation 51 which, under the stationary condition takes the form,

$$\lambda_{3,c} = (1/\alpha)\sqrt{\alpha^2 - (\lambda_{1,c} + \alpha - 1)^2} \tag{102}$$

Using the values of $\lambda_{1,c}$ and $\lambda_{3,c}$ calculated with the aid of Equations 101 and 102, the critical value of diffuser area ratio (δ_c), can be determined with the use of Equation 50 as,

$$\delta_c = \{1 + (\alpha - 1)\lambda_{1,c}\}/\alpha\lambda_{3,c} \tag{103}$$

Using δ_c, $\lambda_{1,c}$, and $\lambda_{3,c}$, the critical value of thrust augmentation can be determined from Equation 99, or using only α, and $\lambda_{1,c}$, the thrust augmentation can be determined from Equation 71, in the form for a stationary ejector which is,

$$\phi = \{1 + (\alpha - 1)\lambda_{1,c}\}(1/\alpha)\sqrt{1 + 2(\alpha - 1)/(1 + \lambda_{1,c})} \tag{104}$$

Using the value of thrust augmentation determined by the above methods, the relative power (R_p) can be determined for the stationary ejector as follows. If it is assumed that $Q_{p,ej} = Q_o$, as in Equation 80, the relative power is

$$R_p = (1/\phi)^2 \tag{105}$$

and if it assumed that $V_{j\infty} = V_{j,o}$, as in the case of Equation 82 then,

$$R_p = 1/\phi \tag{106}$$

A chart is presented on Figure 12 which can be used to determine the thrust augmentation of a stationary ejector by either of two methods.

First, if the jet velocity pressure rise (ΔP) and the depth of operation are given the chart can be entered from the lower left coordinate axes. Proceeding, as illustrated in Example 1 on the chart, to the right until the chosen value of the throat pressure ratio (or to the cavitation limited value of this pressure ratio), then upward to the value of the inlet area ratio (α), where the value of diffuser area ratio (δ), and the thrust augmentation (ϕ), can be read on the appropriate scales.

Alternatively, the values of α and δ can be used to enter the chart at the upper right set of coordinates, giving the value of thrust augmentation, λ_1, and the ratio of primary jet area (a) to reference jet area (a_∞), for those chosen values of α and δ. The evidence of cavitation can then be determined for any desired values of ΔP and depth, by proceeding from the limiting cavitation curve left to the depth and jet velocity pressure rise coordinates, as indicated by Example 2 on the chart.

Having the value of thrust augmentation, the relative power (R_p) can be determined from Equations 105 or 106 or from Figure 13.

Figure 12

PERFORMANCE OF STATIC UNDERWATER EJECTOR WITH SOLID DIFFUSER

FLIGHT DYNAMICS RESEARCH CORP. VAN NUYS, CALIFORNIA

SYMBOLS

- a = primary jet area at throat section
- a_∞ = primary jet area at ambient pressure
- A_2 = throat area
- P_1 = static pressure at throat section
- P_{op} = stagnation pressure of the primary jet
- P_∞ = ambient pressure = 14.7 + 0.4444 × depth (ft), psi
- ΔP = $P_{op} - P_\infty$ = incremental stagnation pressure of the primary jet
- δ = diffuser area ratio
- ϕ = thrust augmentation
- λ_1 = velocity ratio of induced flow to primary flow at throat section

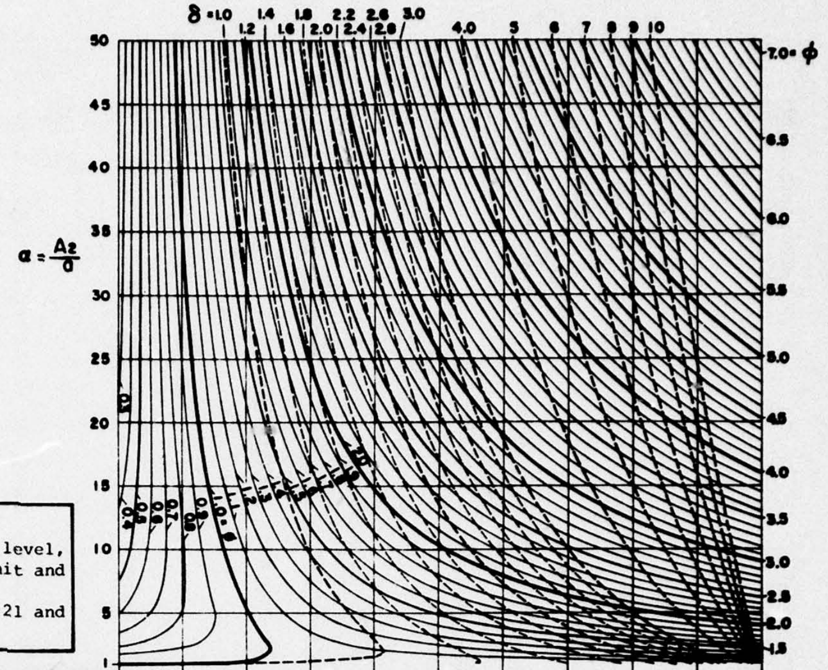
PROCEDURES

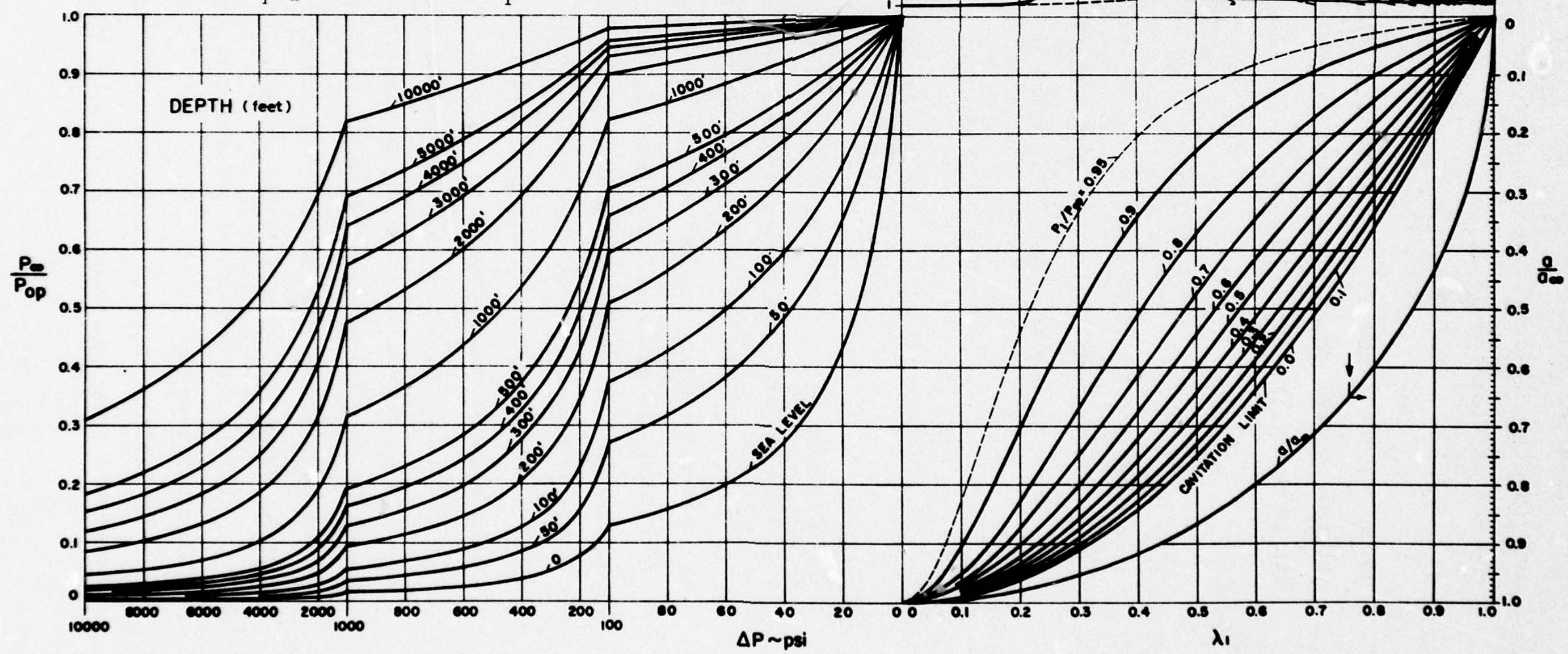
1) $\Delta P \uparrow$ (depth) $\rightarrow P_1/P_\infty \updownarrow$; $\alpha \Rightarrow \phi, \delta$; (curve of jet area ratio) $\rightarrow a/a_\infty$

2) $\alpha + \delta \Rightarrow \phi, \lambda_1$, (curve of jet area ratio) $\rightarrow a/a_\infty$
find relation of P_1/P_∞, depth and ΔP from λ_1

EXAMPLE

ΔP = 30 psi, sea level, at cavitation limit and α = 25, then, ϕ = 3.31, δ = 3.21 and a/a_∞ = 0.819





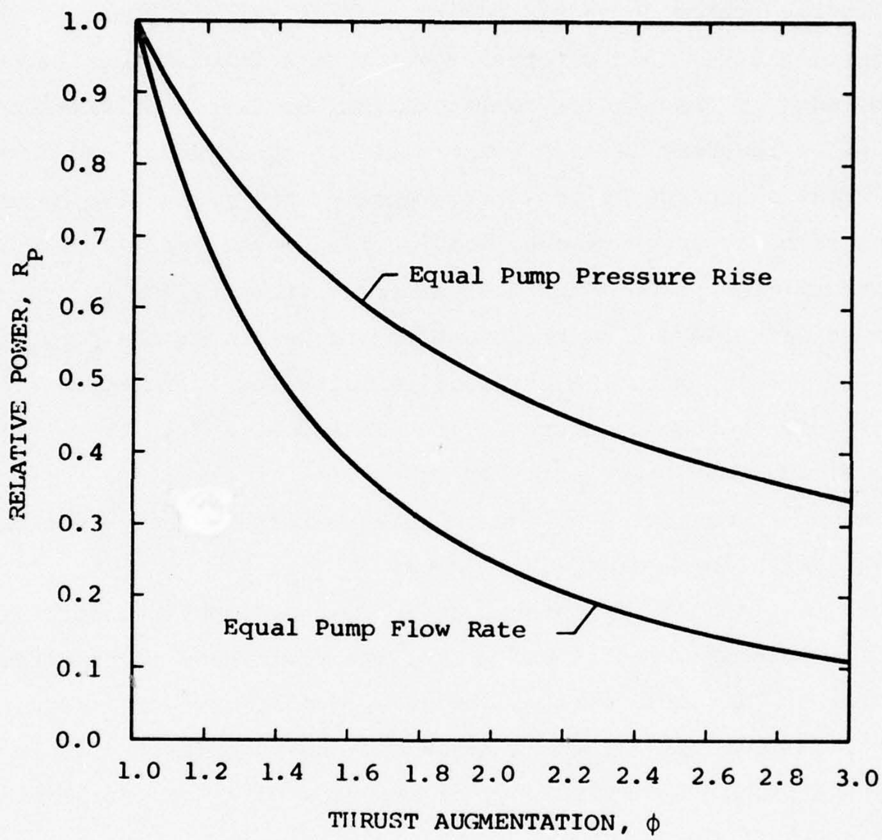


Figure 13 POWER REQUIREMENT OF STATIONARY EJECTOR RELATIVE TO POWER REQUIREMENT OF FREE JET AT EQUAL THRUSTS

2. EJECTOR IN MOTION

The effectiveness of an ejector as the propulsive element of an underwater vehicle in motion, depends upon its net thrust (gross thrust minus ram drag), the additional drag and duct losses resulting from the ejector installation and the skin friction losses within the ejector.

Since the ejector geometry and the ducting arrangement will vary widely among different applications, the drag and duct losses must be evaluated by the system designers during the preliminary phase of the design effort. Internal and external ejector skin friction can be evaluated from a knowledge of the ejector configuration, as discussed previously.

The use of boundary layer or wake fluid in the ejector and/or the pump, can be of great advantage to the performance of the propulsive system, and with proper boundary layer removal, can provide large vehicle drag reduction.

In general, the performance and power requirements for an ejector in motion can be determined from the equations presented in the previous section.

Equations 58 to 61 can be utilized to determine λ_3 in terms of the ejector's geometry, speed, average boundary layer velocity at the inlet of the ejector and its pump, and the jet velocity pressure rise of the pump.

The thrust augmentation at the selected values of the parameters can then be determined with the aid of Equation 68.

The simplest method for determination of the propulsive efficiency then consists of a determination of the propulsive efficiency of the reference jet from Equation 39, and from this the propulsive efficiency of the ejector can be determined from Equation 72, which describes the propulsive efficiency of an ejector as the propulsive efficiency of its reference jet (η_{ref}) times the thrust augmentation (ϕ) of the ejector.

With this information, the relative power can be evaluated by the use of Equation 80 for the case in which the flow rate of the free jet is equal to that of the primary jet of the ejector. Equation 82 can be used for the case in which the jet velocity pressure rise of the ejector's primary jet is equal to that of the free jet.

This method although cumbersome, provides the required information regarding the ideal performance of an ejector under virtually all conditions, but it is essential that the limit imposed upon the diffuser area ratio (δ) by cavitation not be exceeded. To avoid this, the critical diffuser area ratio must be determined in terms of the ejector's geometry and operational conditions.

This critical diffuser area ratio can be determined as follows.

Using Equation 84, determine the critical value of $\lambda_{1,c}$ in terms of the pump inlet velocity (U_p), the ejector inlet velocity (U_e), the jet velocity pressure rise (ΔP), the vehicle's velocity (U_∞), and the depth pressure (p_∞).

The value of $\lambda_{3,c}$ can then be determined from Equation 70, and the critical diffuser area ratio (δ_c) can be determined with the aid of Equation 50.

a) Free-stream Ejector with Fixed Geometry

An ejector operating with fixed geometry (α and δ are constant), at any arbitrary speed and depth, but with its inlet in the free stream, can be analyzed by the general equations (Equations 58 to 61, 68, 72 and 80).

Figures 14 and 15 illustrate the performance criteria of a free-stream ejector with an inlet area ratio (α) of 40 and 100 respectively, as a function of $q_\infty/\Delta P$, and at various values of the diffuser area ratio (δ).

A prominant characteristic of ejectors, easily observed on these figures, is the rapid deterioration of the thrust augmentation and relative power as $q_\infty/\Delta P$ increases.

It is because of this characteristic that it is generally believed that ejectors are not useful at high speeds. Careful thought however, will indicate that regardless of speed (or free-stream dynamic pressure, q_∞), the value of ΔP can be chosen to provide any desired value of the ratio $q_\infty/\Delta P$, thus optimizing the ejector performance as required by the application. The utility of an ejector as a thrust augmenter is therefore not limited by speed alone.

Care must be exercised in the selection of ΔP however, to assure that the phenomenon of cavitation does not occur within the ejector. Excessively high pump pressure rise can result in cavitation at the ejector's throat, as previously discussed.

The onset of cavitation, depends upon the pump pressure rise (ΔP), and upon the depth pressure (p_∞). The limits of the ratio $q_\infty/\Delta P$, at which the ejector can operate are related to the quantities ΔP, p_∞, and to the diffuser area ratio (δ), as indicated on Figures 14 and 15.

Since the inlet area ratio (α) relates the ejector's size to the size of the primary jet, the actual size of an ejector depends upon the primary jet exit area which, for a given thrust, decreases with increasing values of ΔP.

As indicated on Figures 14 and 15, increasing the value of δ requires the use of smaller values of $\Delta P/p_\infty$, if cavitation is to be avoided.

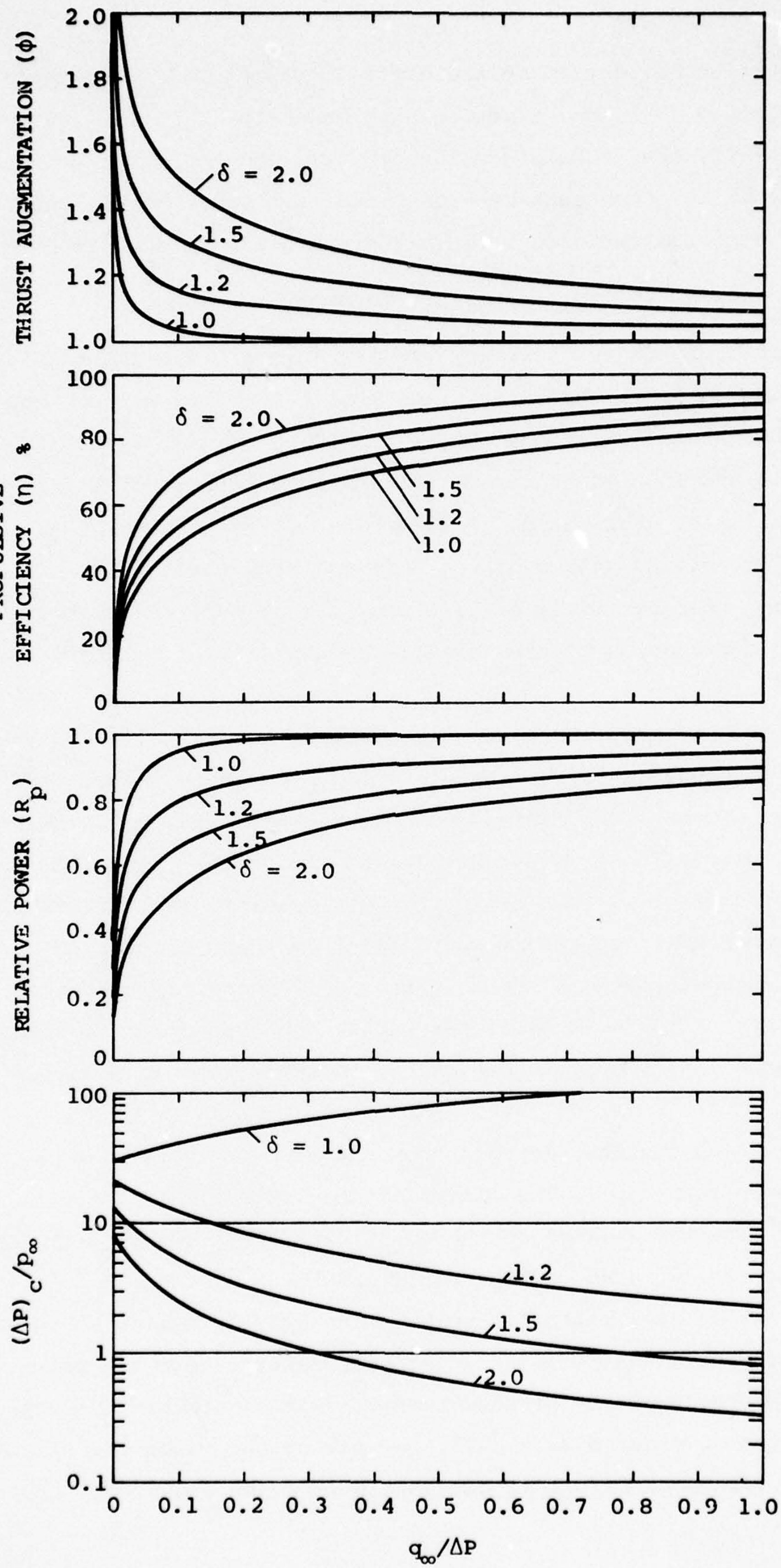


Figure 14 FREE-STREAM EJECTOR PERFORMANCE

$\alpha = 40$

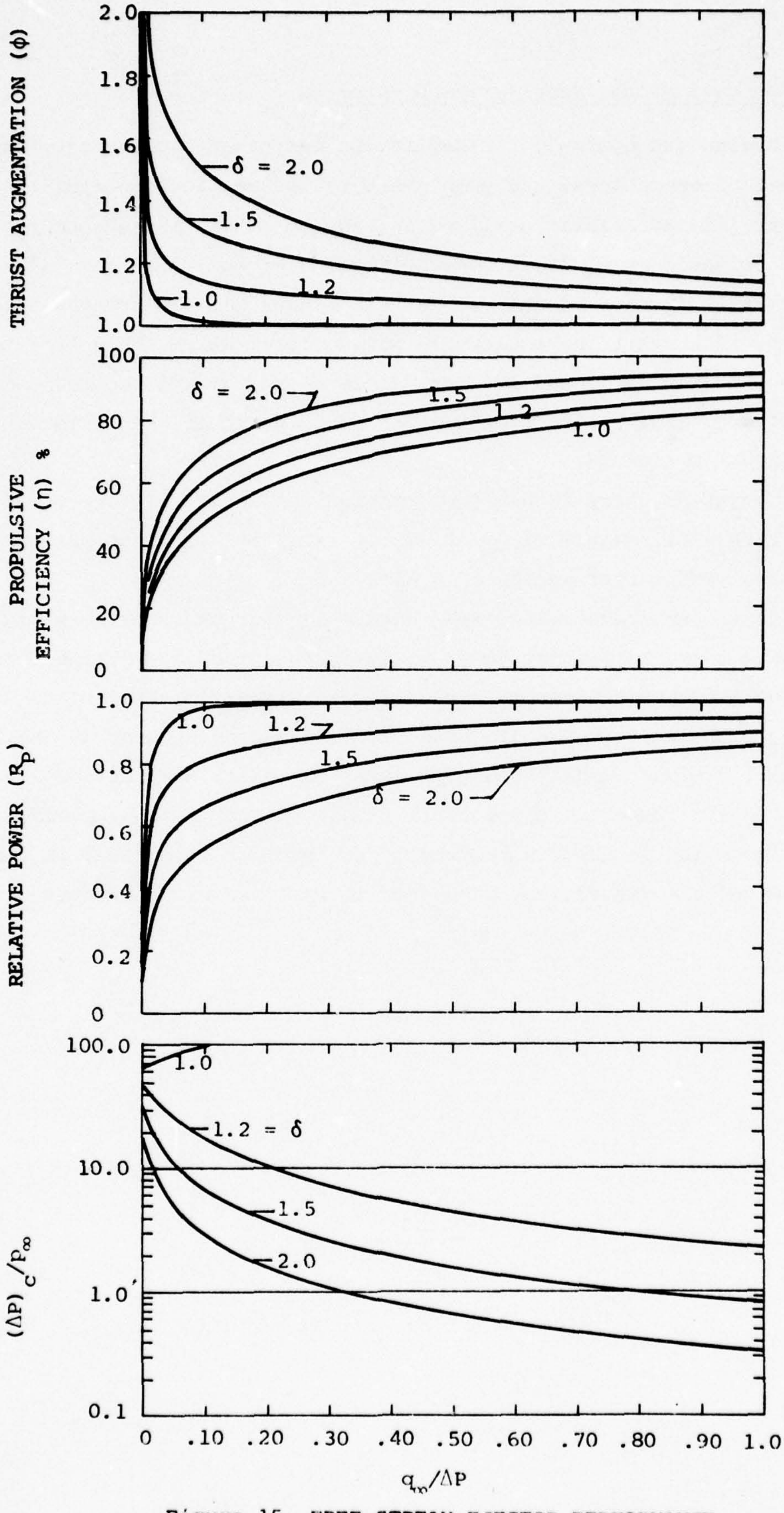


Figure 15 FREE-STREAM EJECTOR PERFORMANCE

α = 100

b) Non-cavitating, Free-stream Ejector

To determine the optimal, non-cavitating performance of an ejector, under any given set of operational and pump characteristics, it is useful to relax the fixed geometry characteristic utilized on Figures 14 and 15, and to plot the performance criteria vs $q_\infty/\Delta P$, at the critical diffuser area ratio ($\delta = \delta_c$), with the value $\Delta P/p_\infty$ as a parameter. In this manner the performance illustrated on the chart, is known to be achievable without cavitation. This type of chart is presented on Figures 16 and 17, for values of $\alpha = 40$ and 100 respectively.

The diffuser area ratios required for achievement of the performance are also presented on these figures.

As illustrated, large thrust augmentation and small relative power are achievable with small values of $q_\infty/\Delta P$, while large values of propulsive efficiency are achievable with larger values of $q_\infty/\Delta P$.

Although the non-cavitating, free-stream ejector can provide a high thrust augmentation and small relative power at small values of $q_\infty/\Delta P$, the performance is limited when the pump pressure rise (ΔP), is large compared to the depth pressure (p_∞). The performance limitations described on Figures 16 and 17, result from the small diffuser area ratios (illustrated on the figures), which are permissible (for a free-stream ejector), if cavitation is to be avoided.

This limitation is greatly relaxed if the ejector is located in the boundary layer or wake of the vehicle, as discussed in later sections of this document.

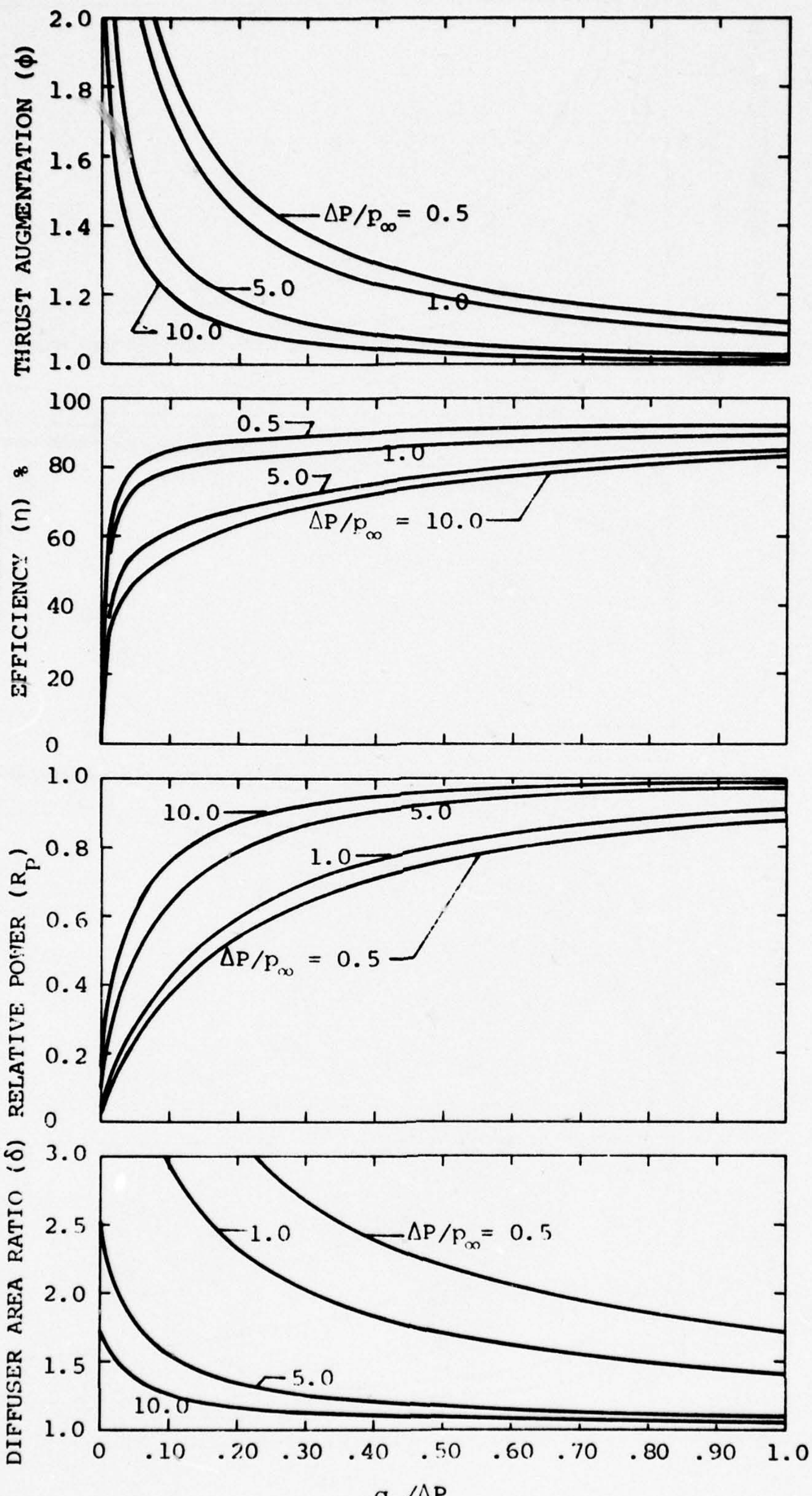


Figure 16 NON-CAVITATING, FREE-STREAM EJECTOR PERFORMANCE AT CRITICAL DIFFUSER AREA RATIO
$\alpha = 40$

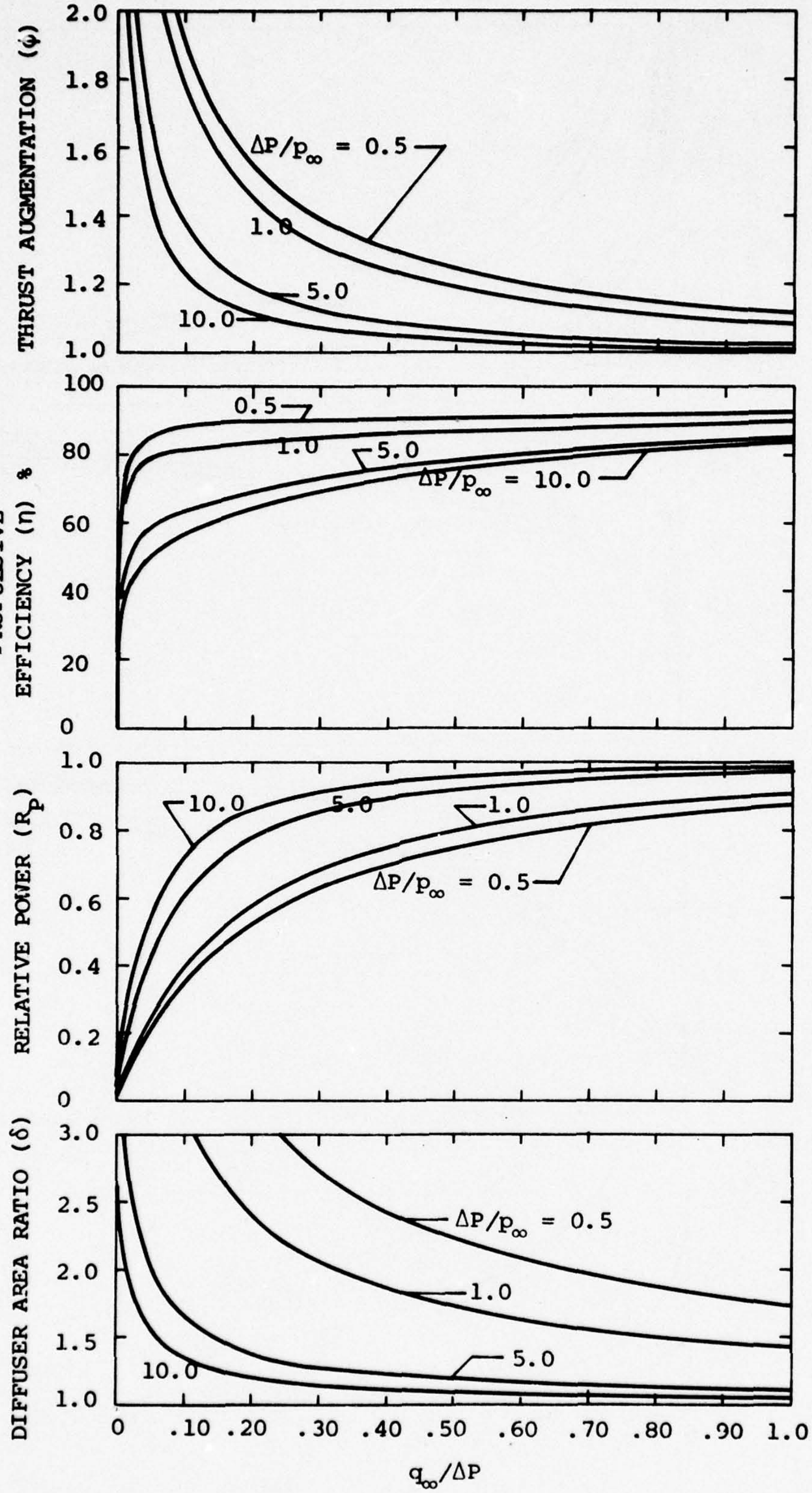


Figure 17 NON-CAVITATING, FREE-STREAM EJECTOR PERFORMANCE AT CRITICAL DIFFUSER AREA RATIO $\alpha = 100$

c) Pump Inlet in Free-stream

Assuming the pump inlet to be in the undisturbed free-stream ($U_p = U_\infty$), while the ejector's inlet is in a region where the average velocity is U_e, provides a basis for determination of the performance criteria and power requirements over any range of ejector inlet velocity ratios ($0 \leqq U_e/U_\infty \leqq 1$).

Under these conditions the ejector's performance can, in general, be determined from Equations 58 to 61, 68 and 72. Its relative power can then be evaluated with the aid of Equation 80.

The thrust augmentation, which is defined as the ratio of the ejector's net thrust to the thrust of a free (unaugmented) jet whose jet power and pump flow rate are equal to those of the ejector, can be determined under the above conditions by first evaluating λ_3 ($= U_3/V_{j1}$) using Equations 58 to 61. These equations, under the prescribed conditions, take the form;

$$\lambda_3 = \{-B + \sqrt{B^2 - 4AC}\} /2A \tag{107}$$

where

$$A = (\alpha - 1)^2 + \delta^2 + \alpha^2\delta^2 \{q_e/(\Delta P)_{eff}\} \tag{108}$$

$$B = 2\delta(\alpha - 2) - 2\alpha\delta \{q_e/(\Delta P)_{eff}\} \tag{109}$$

$$C = -(2\alpha - 3) - \{(\alpha - 1)^2 - 1\} \{q_e/(\Delta P)_{eff}\} \tag{110}$$

and

$$(\Delta P)_{eff} = (\Delta P) + q_\infty - q_e \tag{111}$$

$$q_e = (\rho/2)U_e^2 \tag{112}$$

$$q_p = q_\infty = (\rho/2)U_\infty^2 \tag{113}$$

and therefore

$$q_e/(\Delta P)_{eff} = q_e/\{(\Delta P)\{1 + q_\infty/(\Delta P) - q_e/(\Delta P)\}\} \tag{114}$$

The value of λ_1 can then be determined from the general form of Equation 50 which is

$$\lambda_1 = \{\alpha\delta\lambda_3 - 1\}/(\alpha - 1) \tag{115}$$

Using these values for λ_1, $q_e/(\Delta P)_{eff}$, and $q_\infty/(\Delta P)_{eff}$, the thrust augmentation can be evaluated from Equation 71 in the form,

$$\phi = \frac{\{1+(\alpha-1)\lambda_1\}\left\{\sqrt{\frac{q_e}{(\Delta P)_{eff}} + (1/\alpha)^2\left[1+\frac{2(\alpha-1)}{1+\lambda_1}\right]} - \sqrt{\frac{q_e}{(\Delta P)_{eff}}}\right\} - \sqrt{\frac{q_\infty}{(\Delta P)_{eff}}} + \sqrt{\frac{q_e}{(\Delta P)_{eff}}}}{\sqrt{\{1+q_\infty/(\Delta P)\}\,(\Delta P)/(\Delta P)_{eff}} - \sqrt{q_\infty/(\Delta P)_{eff}}} \tag{116}$$

The propulsive efficiency, which is defined as the ratio of the thrust power to the jet power, and which is therefore applicable to any thrust magnitude, can be evaluated using Equations 39 and 72, in the form applicable to the prescribed conditions which are,

$$\eta_{ref} = 2\sqrt{q_\infty/(\Delta P)}\{\sqrt{1+q_\infty/(\Delta P)} - \sqrt{q_\infty/(\Delta P)}\} \tag{117}$$

and

$$\eta_{ej} = \phi\eta_{ref} \tag{118}$$

The relative power, which is defined as the ratio of jet power required by an ejector, to the jet power required by a free (unaugmented) jet having the same net thrust and pump flow rate as the ejector, can be evaluated with the use of Equation 80 in the form,

$$R_p = 1/\{\phi^2 - (\phi - 1)\eta_{ej}\} \tag{119}$$

The ejector's performance at the critical condition ($\delta = \delta_c$), where the diffuser area ratio is at the maximum value consistent with the avoidance of cavitation, can be evaluated using Equation 84, to determine $\lambda_{1,c}$, which under the prescribed conditions takes the form,

$$\lambda_{1,c} = \sqrt{\{1 + (q_e/\Delta P)(\Delta P/p_\infty)\}/\{1 + (q_\infty/\Delta P)(\Delta P/p_\infty) + \Delta P/p_\infty\}} \tag{120}$$

and where from Equation 70,

$$\lambda_{3,c} = \sqrt{1-\lambda_{1,c}^2}\;\sqrt{q_e/(\Delta P)_{eff} + (1/\alpha)^2\{1 + 2(\alpha-1)/(1+\lambda_{1,c})\}} \tag{121}$$

Using these critical values of $\lambda_{1,c}$ and $\lambda_{3,c}$, the critical value of the diffuser area ratio can be determined from Equation 69 in the form,

$$\delta_c = \{1 + (\alpha - 1)\lambda_{1,c}\}/\alpha\lambda_{3,c} \tag{122}$$

The performance criteria ϕ, η, and R_p can now be determined using Equations 116, 118, and 119, and the critical values of $\lambda_{1,c}$, for an ejector at the critical design configuration.

The large number of parameters which influence the performance of an ejector under the prescribed conditions, preclude the formulation of a generalized chart.

In the most general form, the thrust augmentation, propulsive efficiency, and relative power are dependent upon the ejector's geometry (α and δ), its operational condition (U_∞ and depth), its pump characteristics (ΔP and Q), and the condition of the flow at the ejector's inlet (U_e).

Limiting the discussion and presentation of performance data to the critical condition ($\delta = \delta_c$), results in a reduction of the number of variables. Diffuser area ratio then appears as an implicit variable in the equations for the performance criteria.

Figure 18 presents the thrust augmentation (ϕ), the propulsive efficiency (η), and the relative power (R_p), for an ejector at the critical condition ($\delta = \delta_c$), and for several values of the ejector inlet velocity ratio (U_e/U_∞). This figure is applicable to optimally designed ejectors under the prescribed condition ($U_p = U_\infty$), having an inlet area ratio (α) of 20, and $\Delta P/p_\infty = 1.0$.

The specified values of α and $\Delta P/p_\infty$ represent only one set of assumptions and obviously similar curves can be drawn for other values of these parameters. However, the general dependence of the performance criteria upon $q_\infty/\Delta P$, and the advantage in performance resulting from the use of boundary layer fluid in the ejector is represented on this figure, for discussion.

Under the prescribed conditions ($\delta = \delta_c$; $U_p = U_\infty$), the performance criteria are functions of $q_\infty/\Delta P$, U_e/U_∞, $\Delta P/p_\infty$, and α.

As illustrated on Figure 18, the thrust augmentation and relative power are adversely influenced, while the propulsive efficiency is favorably influenced by increasing values of $q_\infty/\Delta P$.

Since the curves on Figure 18 represent the performance criteria at the critical diffuser area ratio ($\delta = \delta_c$), which decreases with increasing values of $q_\infty/\Delta P$, as illustrated, these trends represent optimal values of the performance under the prescribed conditions.

It is important to note however that as shown on the chart, all aspects of ejector performance (ϕ, η, and R_p) are influenced favorably by the use of boundary layer fluid in the ejector (U_e/U_∞ less than 1.0).

Thrust augmentation, which is the ratio of ejector thrust to the thrust of a free jet having equal jet power and pump flow rate, increases rapidly with decreasing values of U_e/U_∞, as illustrated on Figure 18. The decrease of thrust augmentation with increasing values of $q_\infty/\Delta P$ occurs only for values of U_e/U_∞ in excess of 0.2, for the particular conditions ($\alpha = 20$, and $\Delta P/p_\infty = 1$).

Propulsive efficiency, which is the ratio of thrust power to jet power of the ejector, increases with increasing values of $q_\infty/\Delta P$, and increases more rapidly with the use of boundary layer fluid (U_e/U_∞ less than 1.0), as illustrated on Figure 18.

Relative power, which is the ratio of ejector jet power to the jet power required to drive a free jet of equal thrust and pump flow rate, increases with increasing values of $q_\infty/\Delta P$, but the increase is considerably slower when boundary layer fluid is utilized in the ejector, as illustrated on Figure 18.

All curves on Figure 18 terminate at some finite value of $q_\infty/\Delta P$, except for those representing performance criteria at $U_e = U_\infty$. This termination of the information represents the limiting condition where the ejector's exit velocity (U_3) is equal to the vehicle's velocity (U_∞), or where the net thrust of the ejector is equal to the momentum decrement, or drag, corresponding to the ejector's inlet flow velocity (U_e). In other words, for all points on the curves, the ejector's net thrust is greater than the drag represented by the inlet momentum deficit associated with the value of U_e/U_∞.

Ejector performance as a function of ΔP, at a fixed speed and depth is discussed in the following section.

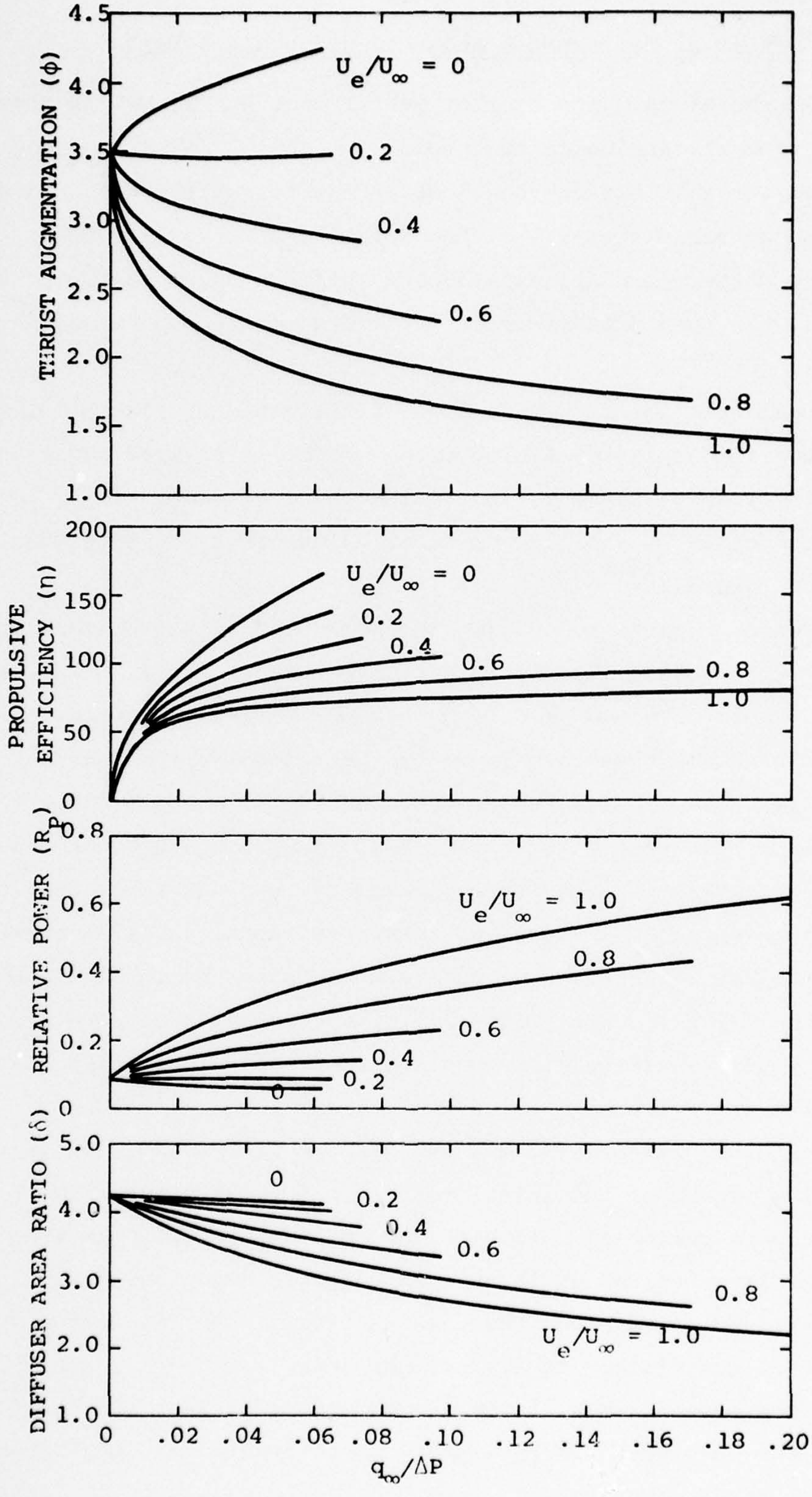


Figure 18 EJECTOR PERFORMANCE IN BOUNDARY LAYER
$U_p = U_\infty$; $\alpha = 20$; $\Delta P/p_\infty = 1.0$; $U_3 > U_\infty$
NON-CAVITATING

1) Ejector Performance at a Fixed Speed and Depth

The influence of ΔP, upon ejector performance can be better understood if separated from the influence of speed.

To illustrate this effect, consider an ejector at the critical diffuser area ratio and at various values of the inlet area ratio (α). Under these conditions the performance criteria depend upon the speed (U_∞), the depth pressure (p_∞), the pump pressure rise (ΔP), and upon the boundary layer velocity ratio (U_e/U_∞).

The special case where $U_e/U_\infty = 0$, is illustrated on Figure 19. As shown, the performance for any given finite value of the inlet area ratio (α), improves with decreasing values of ΔP, to that value where $U_3 = U_\infty$.

The effect of ΔP is similar for other values of U_e/U_∞ less than 1.0, as illustrated on Figure 20.

The curves on Figures 19 and 20 are terminated at the minimal value of ΔP, where $U_3 = U_\infty$, since this limit represents the useful region of the ejector as a propulsive element. Values of $U_3 < U_\infty$ represent design configurations which are useful only as bow thrusters or control elements, since the increment of momentum is less than the drag represented by the injested fluid.

In the case of a free-stream ejector ($U_e = U_\infty$), the ejector's exit velocity is always greater than the free stream velocity, when ΔP is greater than zero. This case is represented on Figure 21, where as shown, no limit exists, and values of ΔP, close to zero can be utilized to drive the ideal ejector.

The optimal diffuser area ratios, limited by cavitation and ejector exit velocity are small, thus avoiding the requirement for large cross-sections to achieve optimal performance.

As indicated on Figures 19, 20, and 21, the performance of ejectors improves with increasing values of the inlet area ratio (α). Although in most applications it is desirable to maintain small size thrust augmenters, it is important to note here that large values of inlet area ratio do not necessarily infer that the ejector will be large since the inlet area ratio (α) indicates only the relative size of the ejector to its primary jet.

In fact the basic thrust of the ejector's primary jet (F_{ref}) increases rapidly with the pump pressure rise (ΔP), or alternatively, the primary nozzle area decreases for a given thrust, with increasing values of ΔP. This results in a small ejector, despite large values of the inlet area ratio (α).

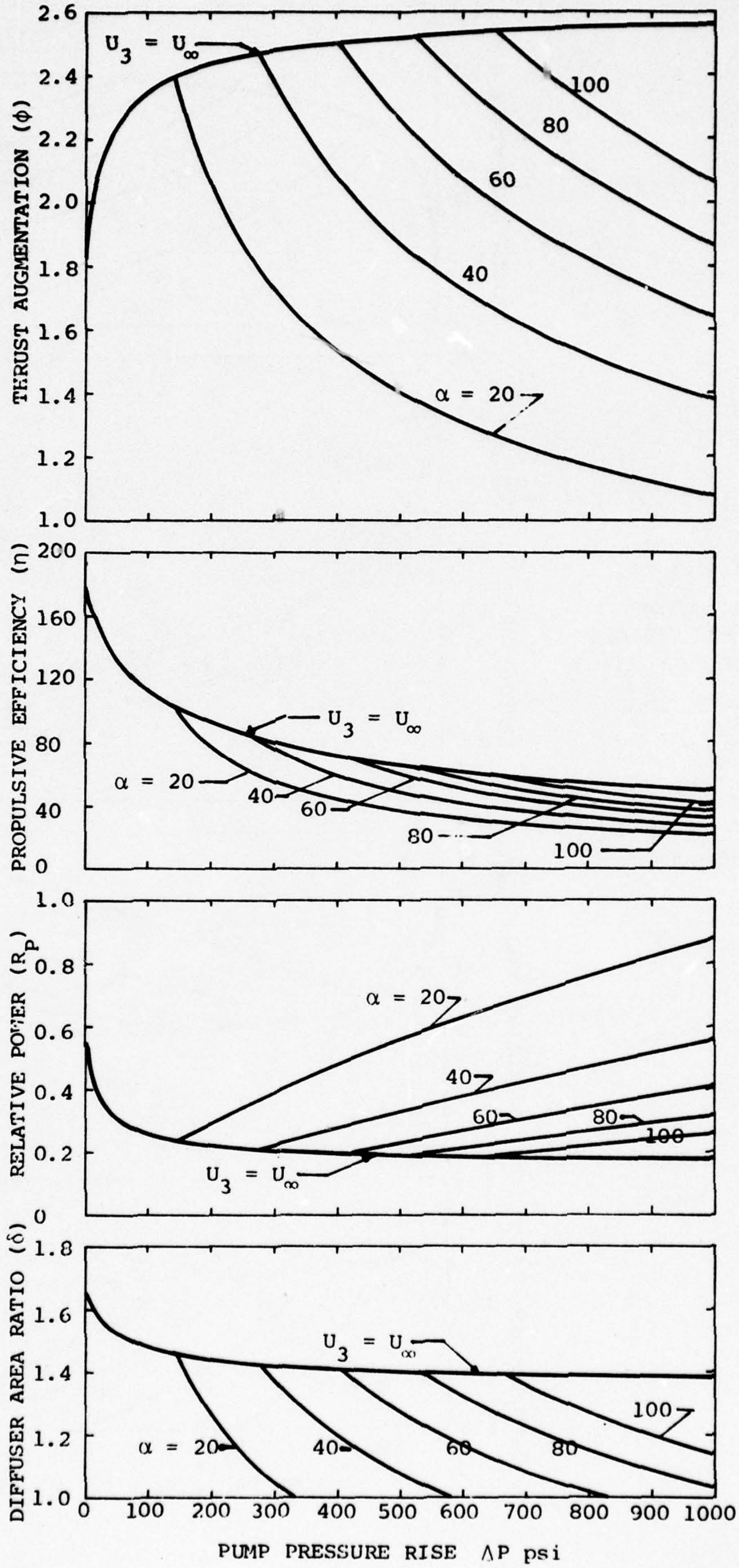


Figure 19

24 KNOTS
SURFACE VEHICLE
EJECTOR PERFORMANCE

U_∞ = 24 Knots

$U_p = U_\infty$

$U_e = 0$

Depth = 12 ft.

NON-CAVITATING

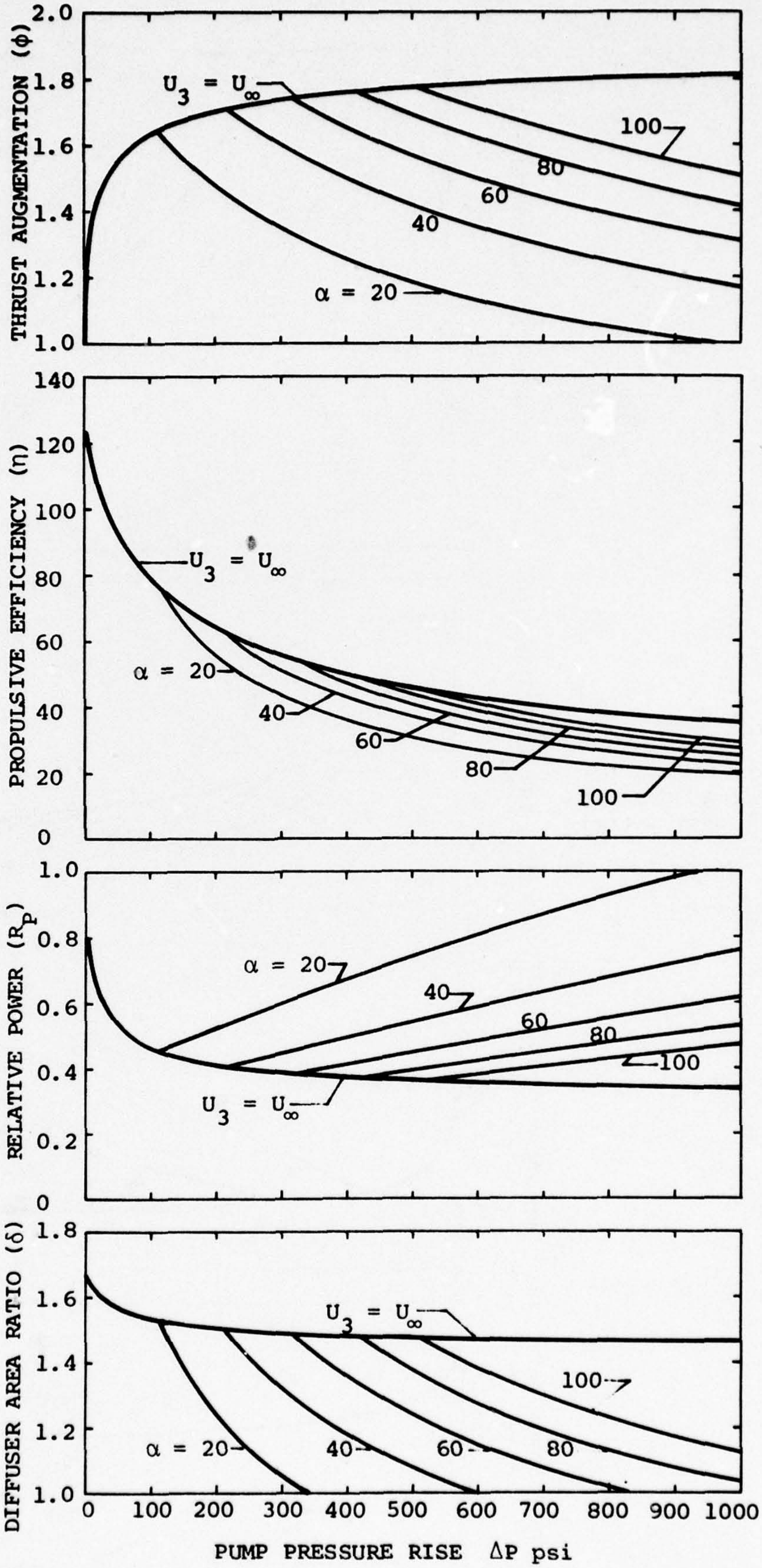


Figure 20

24 KNOTS
SURFACE VEHICLE
EJECTOR PERFORMANCE

U_∞ = 24 Knots

$U_p = U_\infty$

$U_e = 0.5U_\infty$

Depth = 12 ft.

NON-CAVITATING

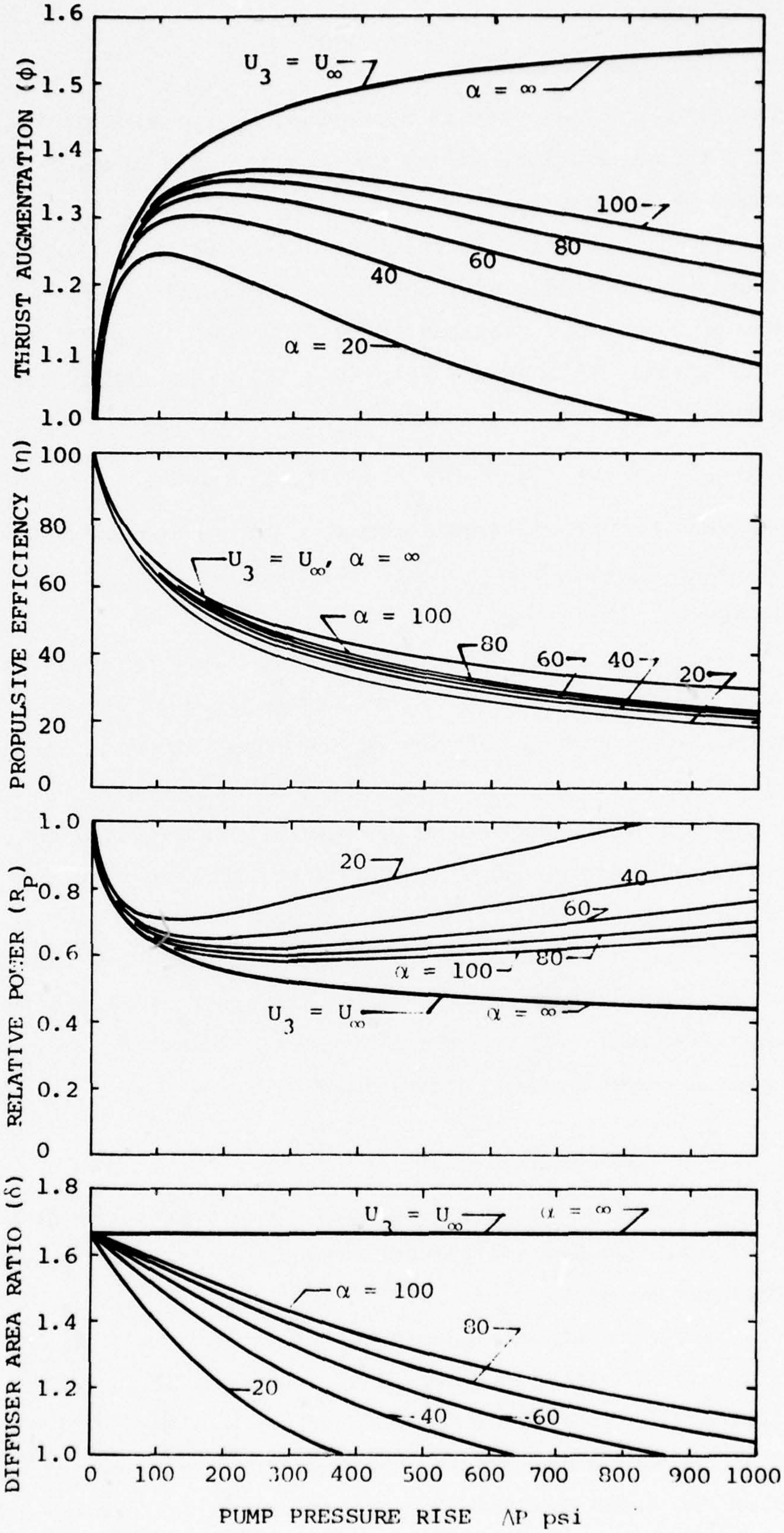


Figure 21

24 KNOTS
SURFACE VEHICLE
EJECTOR PERFORMANCE

U_∞ = 24 Knots

$U_p = U_\infty$

$U_e = U_\infty$

Depth = 12 ft.

NON-CAVITATING

2) Ejector Size

The size of an ejector is determined by the size of its primary jet, at the ejector's throat section, and by the product of α times δ. To illustrate, consider an example as follows:

An ejector operating in the free stream at ΔP = 800 psi, with an inlet area ratio of 100, and a diffuser at the critical value (δ = 1.17) for the operational conditions represented by Figure 21 (U_∞ = 24 knots, depth = 12 ft.)

This ejector will have a reference jet whose thrust is 1430 pounds per square inch of jet exit area since,

$$F/a = 2(q_\infty + \Delta P) - 2\sqrt{q_\infty(q_\infty + \Delta P)} \qquad (123)$$

and therefore the ejector whose thrust augmentation under these conditions is 1.29, will provide a thrust of 1845 lbs with a throat area of approximately 100 sq. in.

The diffuser, in this case has an area ratio of about 1.17, as illustrated on Figure 21, resulting in an overall area ratio of 117 to 1.0. Thus to summarize, the ejector will produce a thrust of 1845 lbs, with its largest dimension equal to only 117 sq. in. or a diameter of about 12 inches.

In addition, the ejector will require approximately 63% as much power as that required by a free jet which produces the same thrust at the same pump flow rate, as indicated by the value of R_p on Figure 21.

Utilization of boundary layer fluid in the ejector will result in even smaller ejectors, requiring less power, as can be observed by carrying out an example similar to that described above, using Figure 20 (where U_e/U_∞ = 0.5), instead of the free-stream ejector of Figure 21.

3) Ejector Drag

The influence of ejector drag upon the performance can be estimated with the aid of Equation 94, and the estimated skin friction drag coefficient (C_D = 0.015) given by Equation 88.

For example, using the ejector characteristics of the above example, the ejector drag, calculated as described (Equations 94 and 88), result in a thrust augmentation (ϕ^*) (corrected for the influence of skin friction) equal to 1.28, instead of the value of 1.29 for the ideal thrust augmentation of the free-stream ejector of the above example. This represents a performance penalty of approximately 1% due to the skin friction of the ejector of the above example.

d) Pump Inlet in Boundary Layer or Wake

The use of boundary layer fluid in the pump of a jet propulsion system introduces a number of practical problems.

In some applications, the fluid must be accumulated from large, widely separated surfaces, requiring large duct distributions between the pump inlet and the pump. In addition, the non-uniform velocity distribution in the boundary layer can create difficult duct design problems, if duct efficiency is to be achieved.

The reduced average velocity of the boundary layer fluid can, in some instances result in cavitation at the first pump stage, thus requiring large pumps if the cavitation is to be avoided.

The analytical problem involved in the determination of the performance criteria under this condition is also somewhat complicated by the introduction of an additional parameter (the average velocity at the pump inlet, U_p). This parameter, if allowed to vary over its entire range (from zero to U_∞), would require a more complex presentation than that of the previous section, where the pump inlet was assumed to be located in the free-stream.

Although the calculation of the ejector performance at any given value of the pump inlet average velocity (U_p) is not difficult with the aid of a computer, the presentation of charts covering the entire range is formidable, and perhaps not warranted in view of the practical difficulties discussed above.

To simplify the presentation problem, and to provide some insight regarding the performance of an ejector propulsion system when the pump and ejector inlets are in a boundary layer, the performance criteria were calculated for the case where the pump and the ejector inlets are at the same velocity ($U_p = U_e = \overline{U}$), and when the ejector is at its critical configuration ($\delta = \delta_c$).

Under the above conditions the value of $\lambda_{1,c}$ is,

$$\lambda_{1,c} = \sqrt{\frac{p_\infty + \overline{q}}{p_\infty + \overline{q} + (\Delta P)}} \qquad (124)$$

where

$$\overline{q} = (\rho/2)\overline{U}^2 \qquad (125)$$

The thrust augmentation can be determined using the value of $\lambda_{1,c}$ derived from Equation 124 with the aid of Equation 71, under these conditions as,

$$\phi = \frac{\{1+(\alpha-1)\lambda_{1,c}\}\{\sqrt{\overline{q}/(\Delta P)+(1/\alpha)^{2}\{1+2(\alpha-1)/(1+\lambda_{1,c})\}} - \sqrt{\overline{q}/(\Delta P)}\}}{\sqrt{1+\overline{q}/(\Delta P)} - \sqrt{\overline{q}/(\Delta P)}} \tag{126}$$

Further, the propulsive efficiency of the reference jet under these conditions is obtained from Equation 39 as,

$$\eta_{ref} = 2\sqrt{q_\infty/(\Delta P)}\{\sqrt{1+\overline{q}/(\Delta P)} - \sqrt{\overline{q}/(\Delta P)}\} \tag{127}$$

and the propulsive efficiency of the ejector is,

$$\eta_{ej} = 2\sqrt{q_\infty/(\Delta P)}\{1+(\alpha-1)\lambda_{1,c}\}\{\sqrt{\overline{q}/(\Delta P)+(1/\alpha)^{2}\{1+2(\alpha-1)/(1+\lambda_{1,c})\}} - \sqrt{\overline{q}/(\Delta P)}\} \tag{128}$$

Under these conditions, the performance criteria are as indicated on Figure 22 where the values of thrust augmentation (ϕ), propulsive efficiency (η), relative power (R_p), are plotted vs $q_\infty/\Delta P$, with the average velocity ratio $\overline{U}/U_\infty$, as a parameter.

The similarity of the performance of the ejector system between the conditions where the pump is in the boundary layer and where the pump is in the free-stream, is illustrated by a comparison of Figure 22 with Figure 18.

Obviously, placing the pump inlet in the boundary layer results in better pump performance, and therefore somewhat smaller thrust augmentation. The propulsive efficiencies of the two systems are virtually the same, and a decision as to which system is superior will depend upon the practical considerations mentioned earlier.

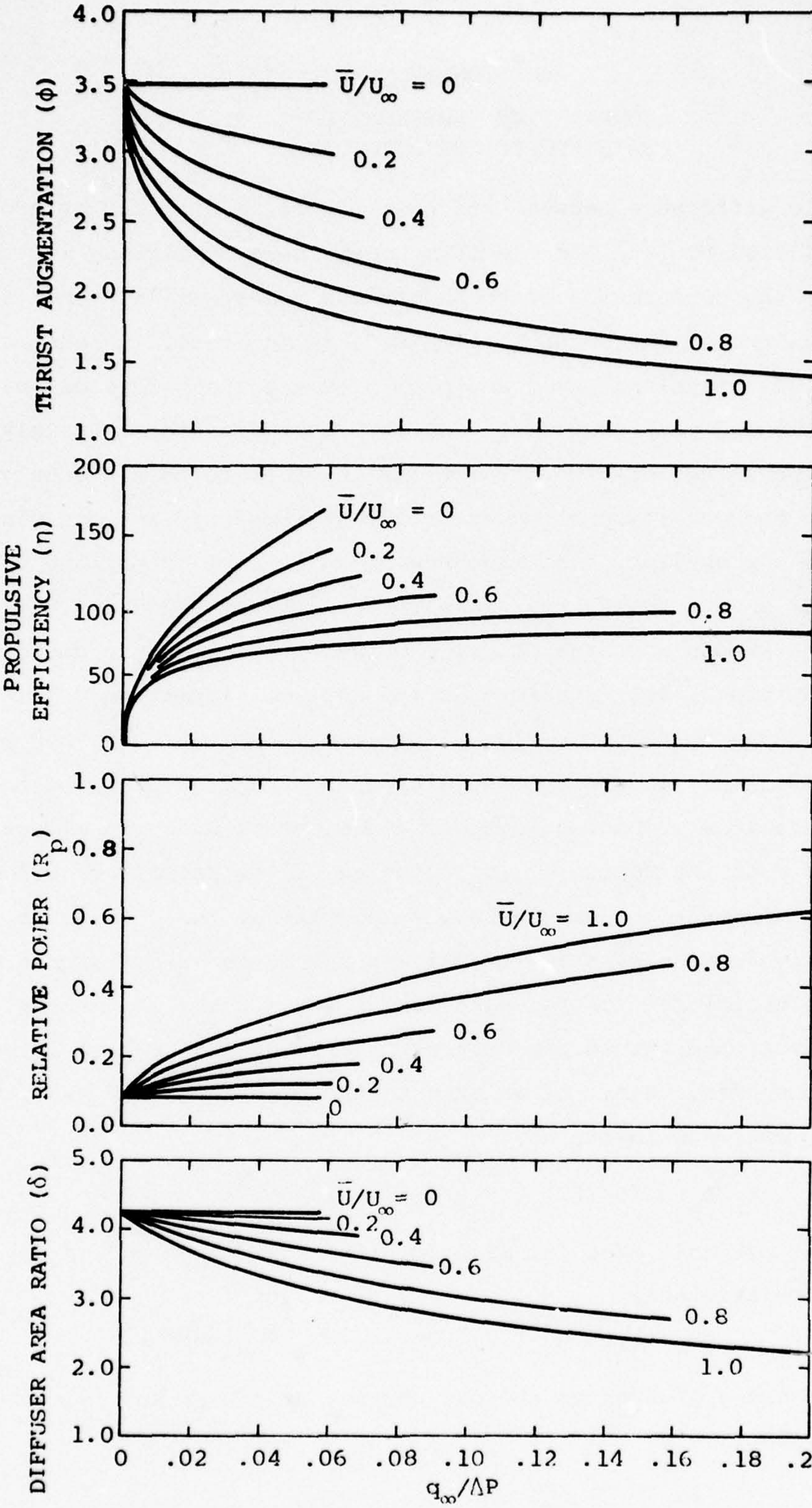


Figure 22 EJECTOR PERFORMANCE IN BOUNDARY LAYER
$U_p = U_e = \bar{U}$; $\alpha = 20$; $\Delta P/p_\infty = 1.0$
$U_3 > U_\infty$; NON-CAVITATING

SECTION V

COMPARATIVE PERFORMANCE JETS/PROPELLERS/EJECTORS

The basic difference between the power supplies utilized by propellers and those utilized by jets and ejectors create some complexity in the comparison of the performance of these devices under similar conditions.

Free jets and ejectors can be compared, as indicated in Section IV, on the basis of equivalent power and pump pressure rise (ΔP), or on the basis of equal thrust and pump flow rate. However, as indicated previously, the comparison always requires two assumptions in order to obtain a unique solution.

Imposing the condition of power or thrust equality, in comparing a jet (or ejector) with a propeller, also requires an additional condition. It is convenient to utilize for this purpose, the propeller slipstream velocity ($V_{j\infty}$) as a parameter.

Since the simple momentum theory provides a relationship among the disc loading, ideal power, free-stream velocity and the slipstream velocity of a propeller, the use of the propeller slipstream velocity ($V_{j\infty}$), can relate the propulsive efficiency of a propeller with that of a free jet or ejector.

A dimensionless comparison of jets and ejectors with propellers can be made using the jet velocity ratio ($V_{j\infty}/U_\infty$) plotted vs the propulsive efficiency (η). Although this comparison does not relate the size of the devices it provides a method for evaluation of their relative performance, and a simple calculation can provide size information for each device, from these parameters.

For example, the thrust per unit area (or disc loading) of a free propeller in a wake or boundary layer, is related to the dimensionless velocity ratio ($V_{j\infty}/U_\infty$), by the simple momentum theory as,

$$F_p/A_p = (\rho/2)U_\infty^2\,\{(V_{j\infty}/U_\infty)^2 - (\bar{U}/U_\infty)^2\} \qquad (129)$$

and the thrust per unit area (or disc loading) for a free jet (F_j/a_j) is given by the relationship,

$$F_j/a_j = \rho U_\infty^2\,\{(V_{j\infty}/U_\infty)^2 - (V_{j\infty}/U_\infty)(\bar{U}/U_\infty)\} \qquad (130)$$

where F_j, a_j, and $V_{j\infty}$ refer to the net thrust, exit area and exit velocity of a free jet respectively.

The thrust per unit area (or disc loading) of an ejector, based upon its throat area (A_2) is given by the relationship,

$$F_{ej}/A_2 = \rho\delta U_\infty^2 \{(U_3/U_\infty)^2 - (U_3/U_\infty)(\bar{U}/U_\infty)\} \tag{131}$$

where U_3 is the exit velocity of the ejector

A_2 is the area at the throat of the ejector

δ is the diffuser area ratio (A_3/A_2)

The use of the throat area (A_2) as a reference for the disc loading of an ejector is justified by the probable use of jet diffusers for future applications. This type of diffuser does not require large solid surfaces, and therefore the maximum dimension of the ejector is close to its throat area.

The evaluation of the magnitude of the term U_3/U_∞, must be accomplished by the methods described in Section III of this document.

Thus a knowledge of the flow upstream of the inlet to the device and the dimensionless exit velocity ratio, permits an evaluation of the size of each device for any given thrust.

Independent of the size considerations, the propulsive efficiency of free jets, propellers, and an ejector with the specific geometry ($\alpha = 20$), and operating with a pump pressure rise (ΔP) equal to its depth pressure (p_∞), at its critical configuration ($\delta = \delta_c$) are compared on Figure 23.

As indicated on Figure 23, the ejector operating at the same value of $V_{j\infty}/U_\infty$, as that of the propeller, and the free jet, has a considerably higher propulsive efficiency than either of the other two devices, over the entire range of values of this dimensionless velocity ratio.

The quantity $V_{j\infty}$, used on Figure 23 represents the slipstream velocity for a propeller, and the jet velocity achieved by expanding the pump flow to ambient pressure for the free jet and for the ejector.

As shown on Figure 23, the use of boundary layer fluid at the inlet of the ejector's pump, limits the lower end of the range of values of $V_{j\infty}/U_\infty$, where the ejector's exit velocity (U_3) can become smaller than the free-stream velocity (U_∞). However, the use of boundary layer fluid provides large gains in the propulsive efficiency, and operating at the proper values of $V_{j\infty}/U_\infty$, can result in values of the propulsive efficiency above 100%, when boundary layer fluid is utilized in the ejector.

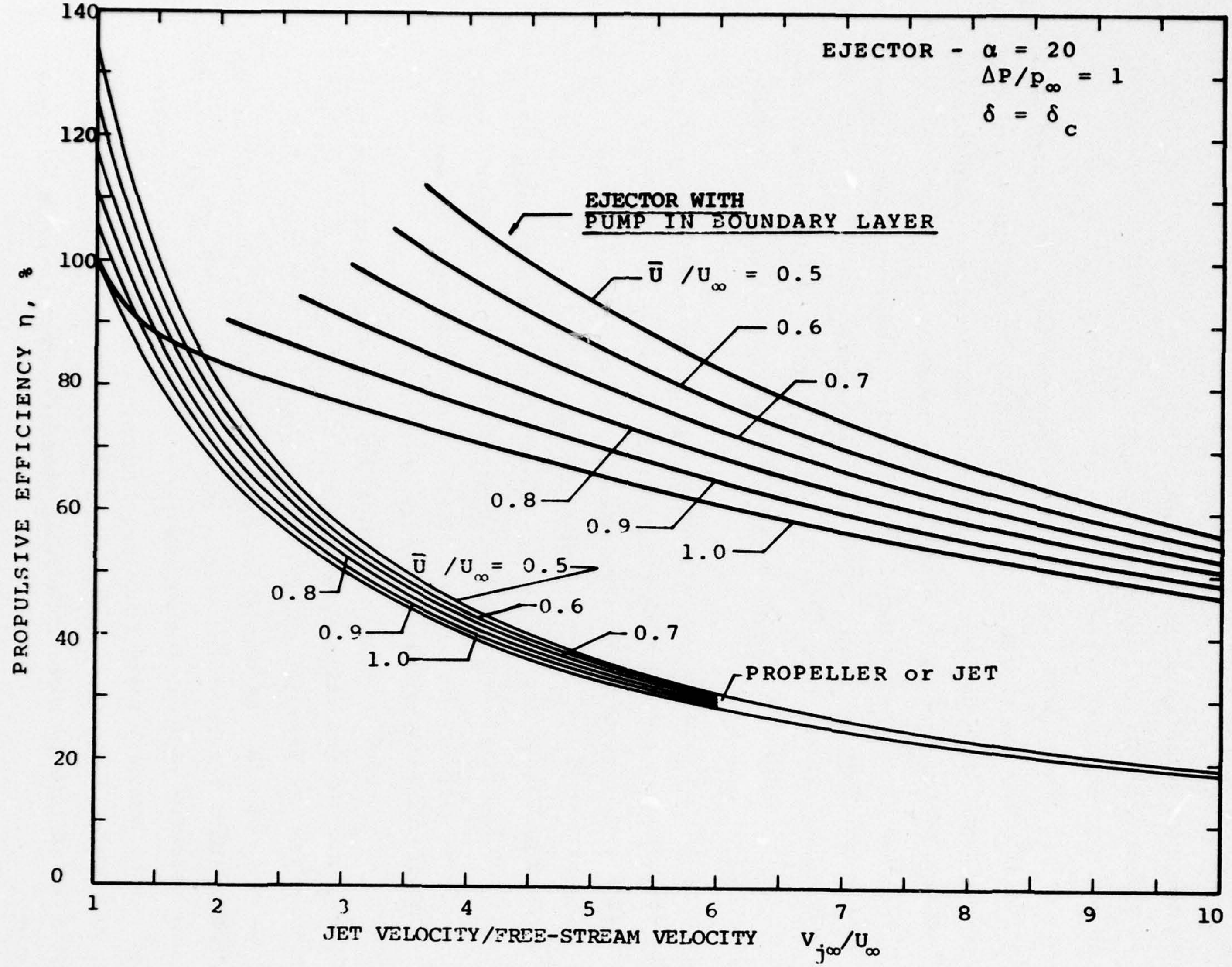


Figure 23

JETS/PROPELLERS/EJECTORS

COMPARATIVE PERFORMANCE

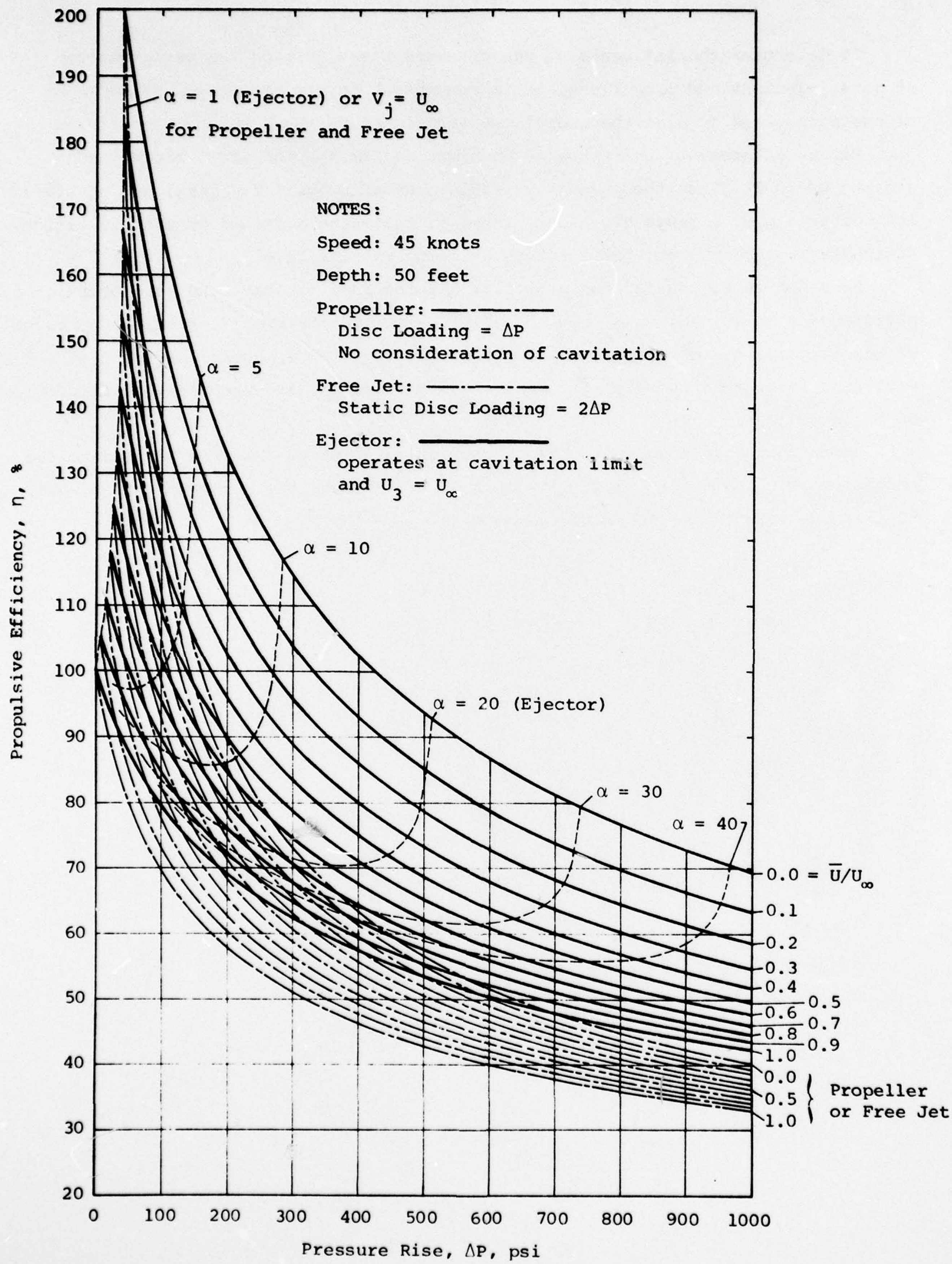


Figure 24. JETS/PROPELLERS/EJECTORS COMPARATIVE PERFORMANCE

To determine the influence of the pressure rise (ΔP), on the performance of jets, ejectors and propellers, it is convenient to fix the speed and depth of operation, and to plot the propulsive efficiency vs the pressure rise (ΔP).

Figure 24 presents this type of information for a speed of 45 knots, and a depth of 50 ft. Thus the propulsive efficiency of propellers, jets, and ejectors are compared over a range of values of ΔP, without consideration of the cavitation limit for propellers, and for ejectors which do not cavitate.

As shown on Figure 24, the propeller and the free jet have the same propulsive efficiencies at any given value of ΔP, but obviously, the propeller cannot operate at the large values of ΔP plotted. The data of Figure 24 is useful however, in providing ideal relationships for the relative values of the propulsive efficiencies of these devices.

As indicated on Figure 24, the ejector is superior to the free jet and to the propeller over the entire range of values of ΔP, despite the relatively high speed of 45 knots assumed in the presented data.

SECTION VI

MULTI-STAGE EJECTORS

In general, multi-stage ejectors are systems of ejectors arranged to influence each other for the purpose of improving their overall performance.

In particular the achievement of high thrust augmentation, can be accomplished by a series arrangement in which each ejector supplies the primary fluid to drive the next ejector of the series.

Arranged in this manner, the thrust augmentation of the system is the product of the thrust augmentations of each ejector in the system.

$$\phi = {}_1\phi \times {}_2\phi \times {}_3\phi \ldots {}_N\phi \qquad (132)$$

where the subscript to the left of the symbol designates the stage number of the ejector in the system.

To avoid excessive complexity, consider a two-stage ejector system, such as is represented schematically on Figure 25. In this case, N = 2, and the thrust augmentation is the product of ${}_1\phi \times {}_2\phi$, and the propulsive efficiency and relative power are determined in a similar manner as for a single stage ejector, using the overall thrust augmentation.

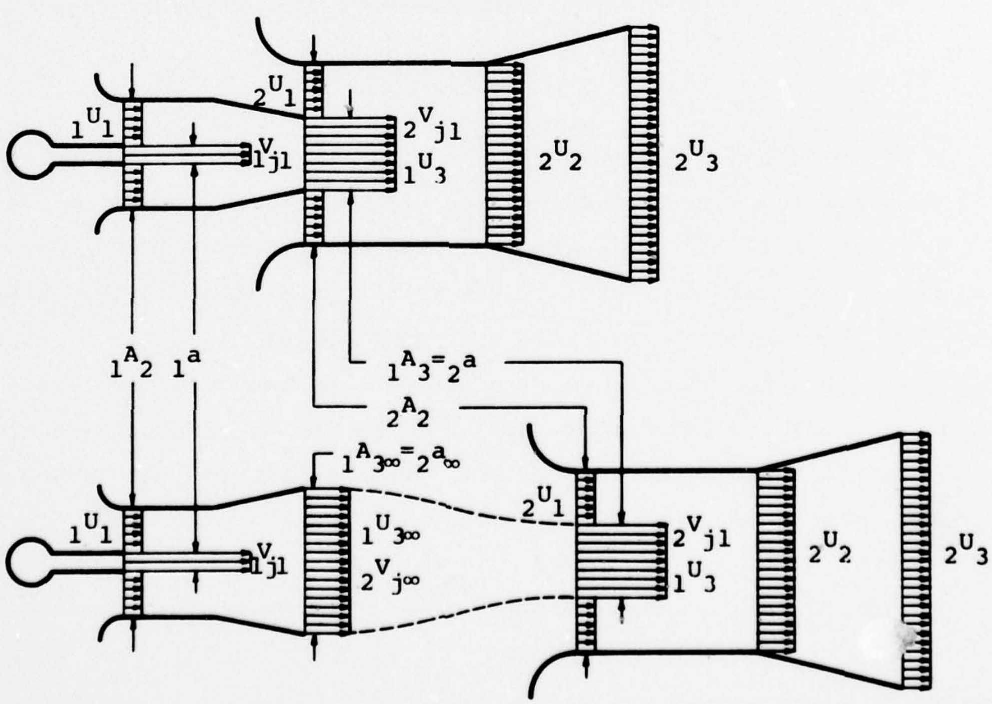


Figure 25. SCHEMATIC REPRESENTATION OF TWO-STAGE EJECTOR

This two-stage ejector system can be considered to be comprised of two free single-stage ejectors, acting in series provided that the analysis correctly accounts for the facts that;

1) The first stage discharges into the throat of the second stage (${}_1p_3 = {}_2p_1$) and

2) The proper value of the second stage pressure rise ${}_2(\Delta P)$ is utilized in the evaluation of its performance.

Since the first stage discharges into a region of reduced pressure, the effective diffuser area ratio (${}_1\delta_\infty$), which would result in a discharge to ambient pressure while maintaining the same pressure at the throat of the ejector, is larger than the geometric diffuser area ratio (${}_1\delta$).

The relationship between the effective and geometric diffuser area ratio of the first stage ejector can be described with the aid of Equations 56 and 67, and the law of mass conservation as,

$$ {}_1\delta/{}_1\delta_\infty = {}_1A_3/{}_1A_{3\infty} = {}_2V_{j\infty}/{}_2V_{j1} = \sqrt{({}_2P_{op} - p_\infty)/({}_2P_{op} - {}_2P_1)} \tag{133} $$

or

$$ {}_1\delta/{}_1\delta_\infty = \sqrt{(1 - {}_2\lambda_1^2)(1 + {}_2q_e/{}_2(\Delta P)_{eff})} \tag{134} $$

where

${}_1A_{3\infty} = {}_2a_\infty$ = Effective diffuser area of first stage

${}_2V_{j\infty} = {}_1U_{3\infty}$ = Effective discharge velocity of first stage.

and

$$ {}_2(\Delta P)_{eff} = {}_2(\Delta P) - {}_2q_e + {}_1q_e \tag{135} $$

As indicated by Equation 133, the effective diffuser area ratio of the first stage (${}_1\delta_\infty$) is larger than the geometric (${}_1\delta$), since ${}_2P_1$ is smaller than p_∞.

In general, if ${}_1\delta_\infty$ is assumed to be given, the solution for the flow parameters of the first stage can be evaluated by the general method for a single stage ejector (Equations 50, 57 - 61, and Equation 68).

The solution of the second stage problem can be accomplished in a similar manner, since the exhaust pressure rise of the first stage, relative to its inlet condition, is the jet velocity pressure rise of the second stage ${}_2(\Delta P)$, which, using Equations 54 and 56 is,

$$ {}_2(\Delta P)/{}_1(\Delta P)_{eff} = \frac{(\rho/2)\,{}_1U_{3\infty}^2 - {}_1q_e}{{}_1(\Delta P)_{eff}} = \frac{{}_1\lambda_{3\infty}^2}{(1 - {}_1\lambda_1^2)} - \frac{{}_1q_e}{{}_1(\Delta P)_{eff}} \tag{136} $$

since

$$ {}_1(\Delta P)_{eff} = (\rho/2)\,{}_1V_{j1}^2(1 - {}_1\lambda_1^2) \tag{137} $$

Equation 136 can be further simplified using Equation 70, or,

$$_2(\Delta P)/_1(\Delta P)_{eff} = (1/_1\alpha^2)(1 + 2(_1\alpha - 1)/(1 + {_1\lambda_1})) \tag{138}$$

Using this value of $_2(\Delta P)$, where the average pump intake velocity is taken as $_2U_e$, the value of $_2\phi$ can be established from Equations 50, 57 - 61, and Equation 68, as was done for the first stage.

The solution of the problem when both stages are at their critical geometry, ($_1\delta = {_1\delta_c}$ and $_2\delta = {_2\delta_c}$) is more easily obtained.

In this case,

$${_1\lambda_{1,c}} = \sqrt{(p_\infty + {_1q_e})/(p_\infty + q_p + {_1(\Delta P)})} \tag{139}$$

and

$${_2\lambda_{1,c}} = \sqrt{(p_\infty + {_2q_e})/(p_\infty + {_1q_e} + {_2(\Delta P)})} \tag{140}$$

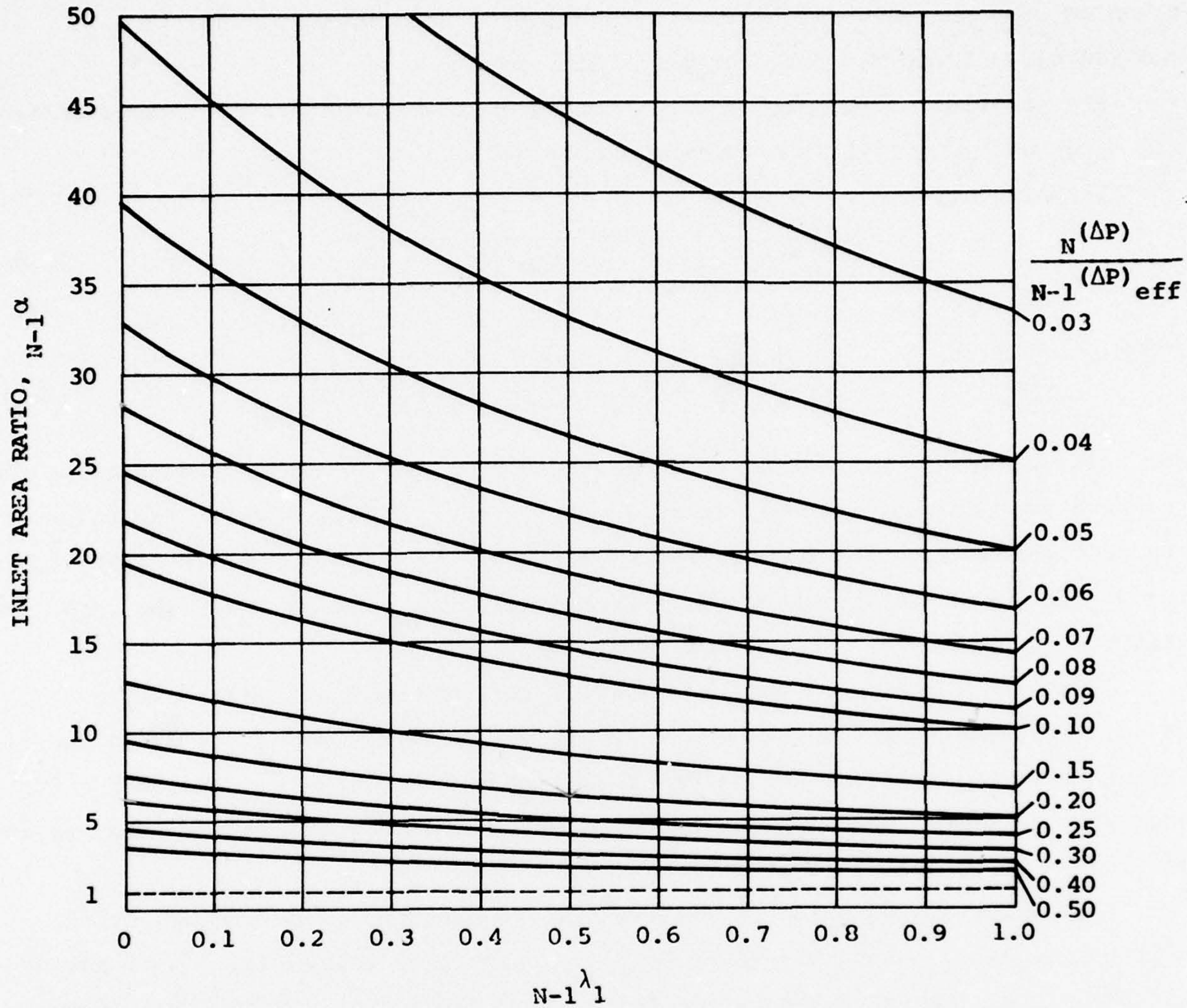
and using Equation 71, the thrust augmentations $_1\phi$ and $_2\phi$ can be evaluated using these values and the given values of q_p, $_1q_e$, $_2q_e$, $_1(\Delta P)$ and $_2(\Delta P)$. The values of the diffuser area ratios are not required in this solution since, as discussed, the diffuser area ratios enter the solution only implicitely, under the condition that the ejector is at the critical geometry.

Using the value of $_1\lambda_{1,c}$ from Equation 139, the value of $_2(\Delta P)$ can be calculated using Equation 138, or by use of Figure 26 for any given value of $_1\alpha$. With this value for $_2(\Delta P)$, Equation 140 can be used to find $_2\lambda_{1,c}$ and the thrust augmentation of the second stage can then be determined from Equation 71, for any desired value of its inlet area ratio (α).

The thrust augmentation of the first stage can then be determined at its critical geometry, using the value of $_1\lambda_{1,c}$ and $_1(\Delta P)$, using Equation 71 again, and the overall thrust augmentation is the product of the augmentations of the two stages.

1. <u>STATIONARY EJECTORS</u>

The solution for the thrust augmentation of a two-stage stationary ejector can be accomplished by the method described above using the fact that $q_e = q_p = 0$. Then using the correct values of $_1(\Delta P)$, $_2(\Delta P)$, $_1\delta_\infty$, and $_2\delta$, the value of the thrust augmentations $_1\phi$, and $_2\phi$ can be evaluated with the aid of Equations 95 to 100 for the general case, or using Equations 101 to 104 for the critical design.



$$_N(\Delta P)_{eff} = {_N(\Delta P)} - {_Nq_e} + {_{N-1}q_e}; \quad {_0q_e} = q_p$$

Figure 26. RATIO OF INTERSTAGE PRESSURE RISE MULTI-STAGE EJECTOR

Evaluation of the performance of a two-stage, stationary ejector can be accomplished as in the following example. The characteristics of the ejector are;

${}_1\alpha$	20.0
${}_2\alpha$	10.0
${}_1(\Delta P)$	200.0 psi
U_∞	0.0
depth	sea level (p_∞ = 14.7 psi)

Assuming the first stage is at its critical diffuser area ratio, the value of ${}_1\lambda_{1,c}$ (=0.2617 for the above example) can be determined using Equation 139 (or Figure 12) and the above values of the required parameters. Using this value for ${}_1\lambda_{1,c}$ and the inlet area ratio ${}_1\alpha$, the thrust augmentation of the first stage ${}_1\phi$ can be determined with the aid of Equation 104, or by using Figure 12.

The value of ${}_2(\Delta P)$ (= 15.5595 for the above example) can then be determined with the aid of Equation 138, or by using Figure 26. Equation 140 can then be utilized to determine the value of ${}_2\lambda_{1,c}$ (= 0.6970 for the above example), which then can be used in Equation 104 to evaluate ${}_2\phi$.

The diffuser area ratios are implicitely assumed to be at their critical values, which can be determined with the aid of Equations 102 and 103, or by using Figure 12, with the proper values of ${}_1(\Delta P)$ and ${}_2(\Delta P)$. The relationship between ${}_1\delta$ and ${}_1\delta_\infty$ is given by Equation 134.

The results of these calculations are listed below.

${}_1\lambda_{1,c}$	0.2617
${}_1\phi$	1.6656
${}_1\delta_{\infty,c}$	1.1091
${}_2(\Delta P)$	15.5595
${}_2\lambda_{1,c}$	0.6970
${}_2\phi$	2.4778
${}_2\delta_c$	2.9770
${}_1\delta_c$	0.7953
ϕ	4.1270

The thrust augmentation of a single-stage ejector having the same pump pressure rise and the same ejector exit area would be 4.3259, slightly larger than the thrust augmentation of the two-stage ejector (ϕ = 4.1270). However, the single stage ejector would require an inlet area ratio of 167.11, and a diffuser area ratio equal to 2.8337.

2. MULTI-STAGE EJECTORS IN MOTION

The analysis of the performance of a multi-stage ejector in motion in its thrust direction can be accomplished in the same manner as described for a single stage ejector, provided the correct values of the diffuser area ratios ${}_N\delta_\infty$, and the pump pressure rises ${}_N(\Delta P)$, are used for each stage.

At the critical design configuration (${}_N\delta_\infty = {}_N\delta_{\infty,c}$) the stage performance can be evaluated using Equation 71, for any given values of ${}_Nq_e$, ${}_{N-1}q_e$ and ${}_N(\Delta P)$, since ${}_0q_e = q_p$. Using the chosen values of these parameters, the value of ${}_N\lambda_{1,c}$ can be evaluated using Equation 84, and the thrust augmentation of the stage can then be determined with the aid of Equation 71.

This procedure was utilized to calculate the thrust augmentation of a series of two-stage ejectors having the identical diffuser exit areas, operational conditions and power supply characteristics. The chosen characteristics are listed below.

$U_\infty = U_p = {}_1U_e = {}_2U_e$	24 knots
depth	12 ft. (p_∞ = 20 psi)
${}_1(\Delta P)$	300 psi
$({}_1\alpha)({}_2\alpha)({}_1\delta)({}_2\delta)$	500

The diffuser area ratios ${}_1\delta$, and ${}_2\delta$ were chosen to be at the critical limit to assure the absence of cavitation.

Varying ${}_1\alpha$, over a range from 1 to 317 (the value at which the second stage inlet area ratio becomes unity), provided the performance and geometric data shown on Figure 27.

As indicated, the thrust augmentation reached a maximum (ϕ = 1.43) at the extreme ends of the range of ${}_1\alpha$ (which are equivalent to the single-stage), and decreased to a minimum (ϕ = 1.32) at intermediate values of the inlet area ratios.

Thus for a given ejector exit area ratio, it is desirable to use a single stage ejector for the achievement of maximum thrust augmentation. Practical considerations however may dictate the use of the multi-stage ejector, since the separation of the stages provide additional flexibility in the orientation and location of the ejectors.

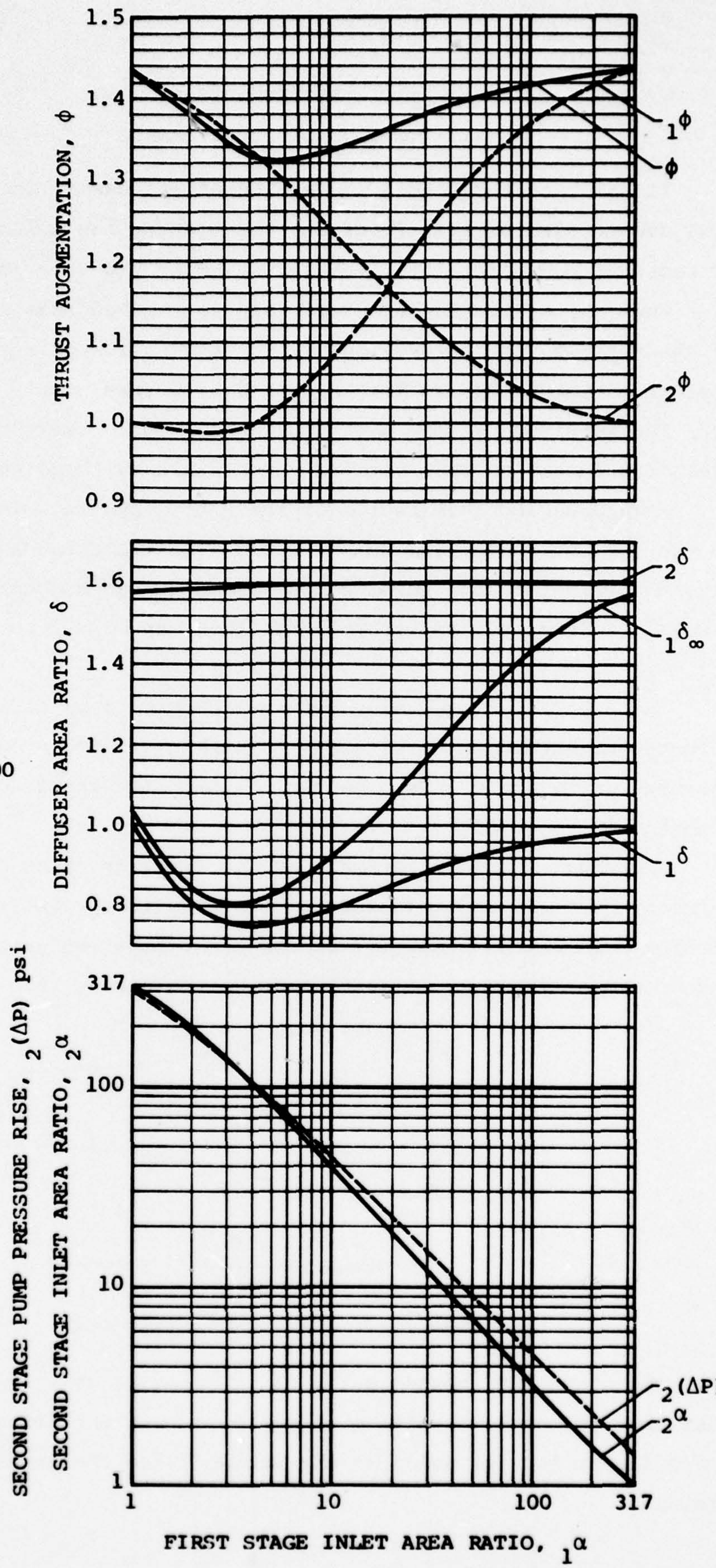


Figure 27

24 KNOTS
SURFACE VEHICLE
TWO-STAGE
EJECTOR PERFORMANCE

U_∞ = 24 Knots

$U_p = {}_1U_e = {}_2U_e = U_\infty$

Depth = 12 ft.

$_1(\Delta P)$ = 300 psi

$({}_1\alpha)({}_1\delta)({}_2\alpha)({}_2\delta) = 500$

NON-CAVITATING

SECTION VII

CONCLUSIONS and REMARKS

It has been shown that, on a theoretical basis, including the additional drag due to ejector skin friction, the ejector has a large advantage in terms of required power for any given thrust, over the free jet and the propeller.

When the ejector's injested fluid is removed from a boundary layer (or wake) of the vehicle, the power required for production of any given thrust is even smaller in comparison to that required by a free jet.

The magnitude of the advantage which would result from the use of an ejector thruster is only partially described by these considerations.

More detailed examination of the entire system, with consideration given to the component size and weight would illustrate the additional savings in the size and weight of the power supply system of an ejector in comparison to the systems of free jets and propellers. This can be discussed in general terms as follows.

Fundamentally, the thrust of any reaction type thruster is the result of a change of the momentum flux of the working fluid. Propellers, free jets, and ejectors differ only in the manner in which the change of the working fluid momentum is achieved.

A propeller applies a force to the fluid by means of a relative motion between the fluid and the blade of the propeller, generally, but not always, without a surrounding duct. Thus the free propeller produces a streamline pattern and pressure distribution as illustrated on Figure 28.

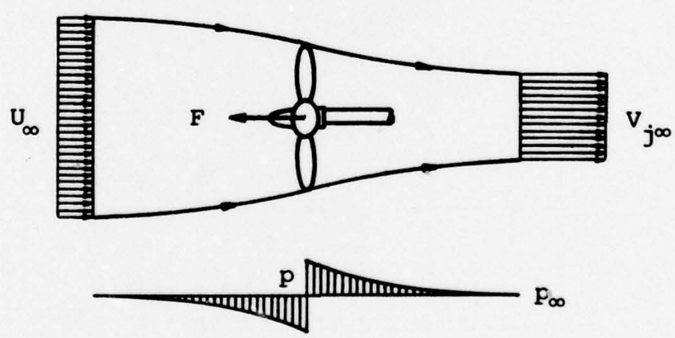


Figure 28. FREE PROPELLER AND ITS PRESSURE DISTRIBUTION

The propeller increases the fluid pressure at its disc, and the fluid stream must then converge, resulting in the fact that the propeller is larger than the effluent jet (or wake) at the region where the pressure has returned to ambient.

A free jet discharges directly to ambient pressure, having a pump which at some upstream location, imparts the force to the fluid, as illustrated on Figure 29.

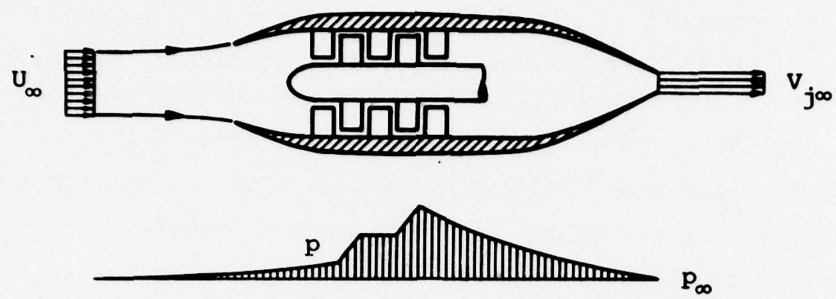


Figure 29. FREE JET SYSTEM AND ITS PRESSURE DISTRIBUTION

The pump, in general, is also larger than the effluent jet, at the region where the pressure has returned to ambient, as illustrated on Figure 29. However, the pump size is smaller than that of the propeller for any given thrust. This is a direct consequence of the fact that the jet can use larger pressure rise than is feasible with a free propeller, without cavitation or stalling.

An ejector, which utilizes a jet as its source of energy, is illustrated on Figure 30, along with its associated pressure distribution.

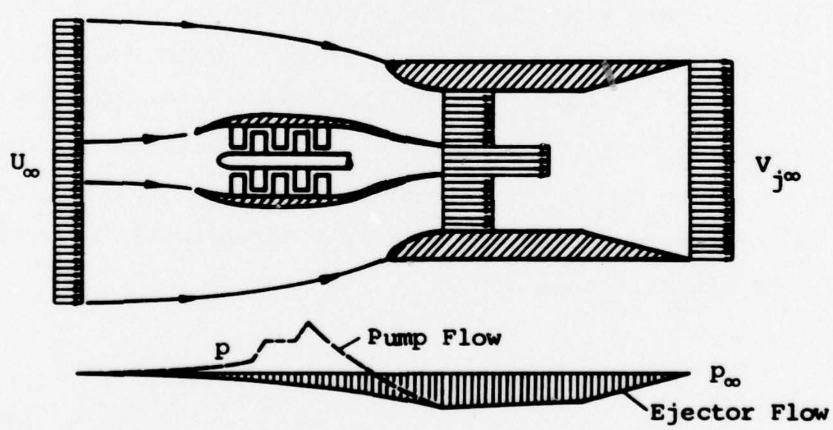


Figure 30. EJECTOR SYSTEM AND ITS PRESSURE DISTRIBUTION

Since the ejector's discharge (at ambient pressure) is a mixture of the primary jet fluid and a large quantity of fluid induced to flow through the ejector, the effluent fluid has a velocity smaller than that of the primary jet velocity. This reduced discharge velocity is accompanied by an increased flow rate, by an amount equal to the induced flow rate. Thus the ejector's pump may operate at a larger pressure rise, and smaller flow rate than that of a free jet producing the same thrust.

Since the ratio of thrust to power, of any thruster which utilizes the environment as working fluid, is given by the relationship, (Equation 5),

$$F/P_j = 2/(V_{j\infty} + U_\infty)$$

where in this case,

$V_{j\infty}$ = the effluent velocity of the thruster, at the region where the working fluid has returned to ambient pressure.

P_j = the effluent jet power of the thruster.

it is evident from the foregoing discussion that the ejector's primary jet may have a larger velocity (which corresponds to a larger pressure rise) than either the propeller or the free jet, for any given value of the ratio of thrust to power. Conversely, for any given pressure rise, the ejector's effluent velocity can be smaller than that of the propeller or the free jet, and therefore with proper design, the ratio of thrust to power of the ejector can be larger than that of a propeller or a free jet.

Thus at any given thrust, an ejector system can be designed to operate at smaller power, and smaller energized flow rate, than either a free jet or a propeller. Both of these factors (power and flow rate) contribute to the size and weight reduction of the ejector system compared to the size and weight of the other systems.

To achieve this advantage, the ejector itself must be larger than the free jet, and in some instances larger than a propeller. This is very little consequence however, in view of the fact that the ejector has no moving parts, and can be tailored to the contours of the vehicle. Thus the penalty in terms of weight and drag are minimal, and as indicated, the net result of the use of ejector thrust is that of a considerable reduction in the system size, weight and fuel consumption.

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